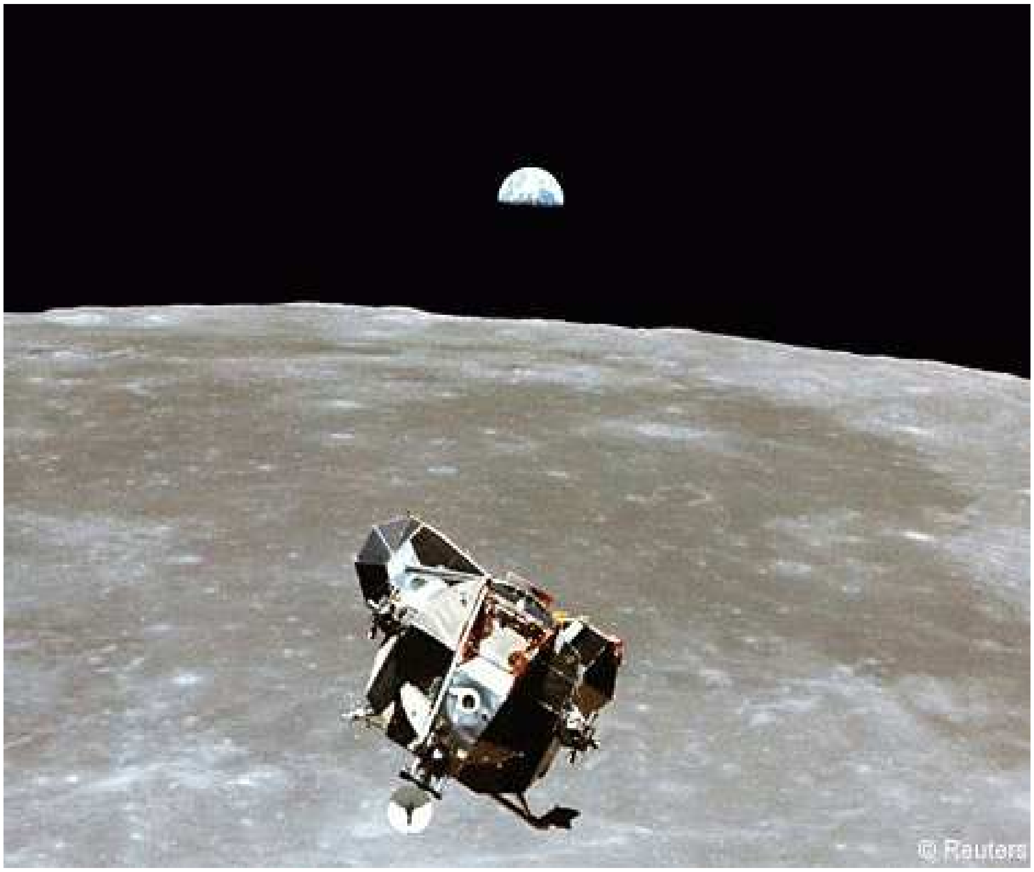
Mathematicians look at particle physics

Matilde Marcolli

"Year of Mathematics" talk – July 2008

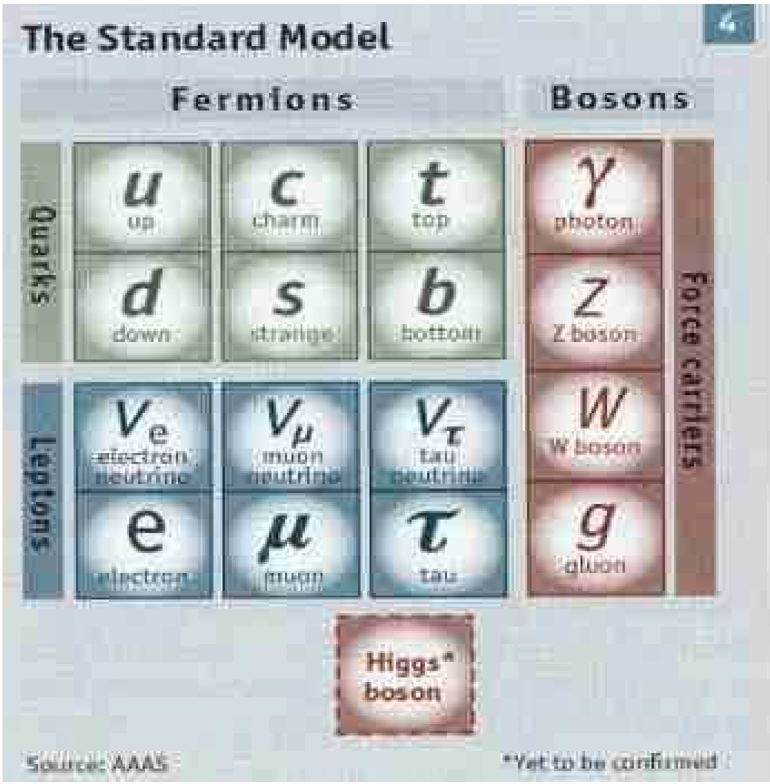


We do not do these things
because they are easy.

We do them because they are hard.

(J.F.Kennedy – Sept. 12, 1962)

Elementary particle physics

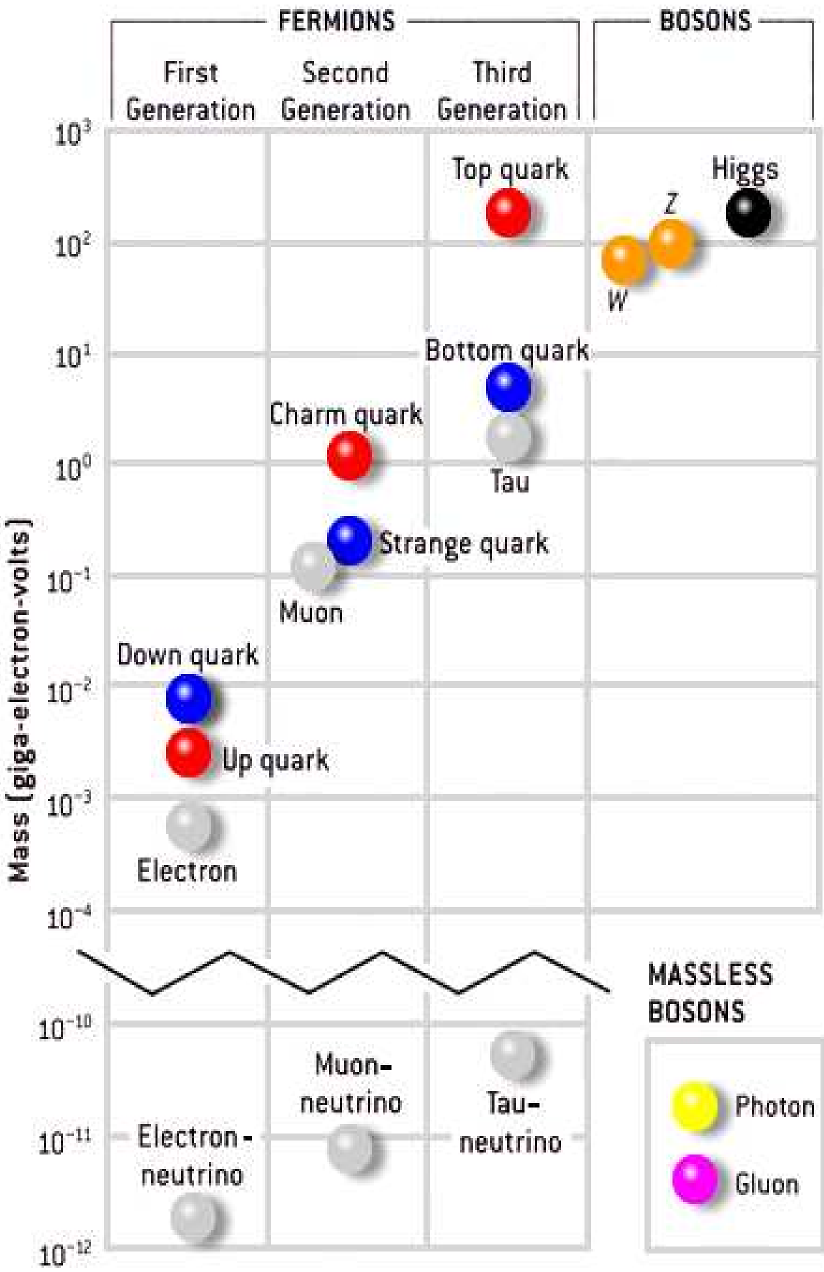


Constituents of all known matters and forces (except gravity)

- Is there new physics beyond?
(massive neutrinos; supersymmetry? dark matter? dark energy?)

- Unification with gravity?
(loops? strings? branes? noncommutative spaces?)

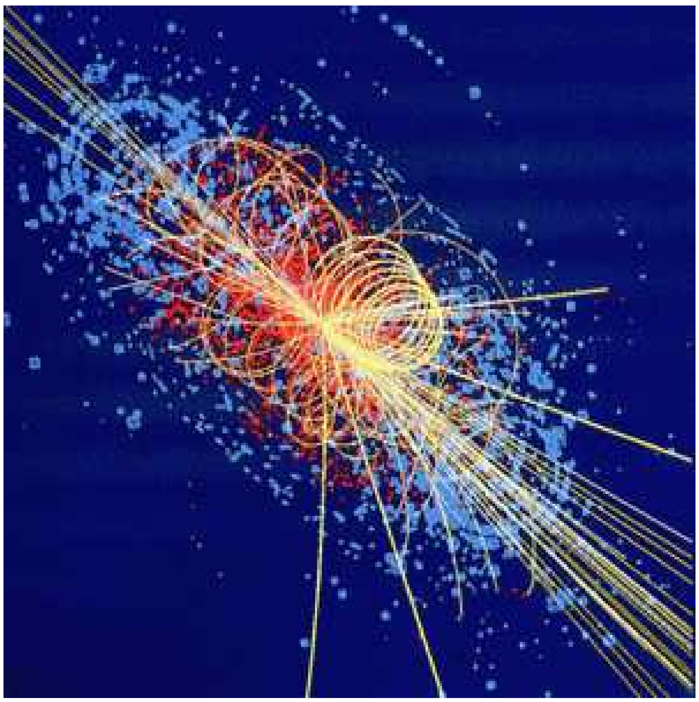
Parameters of the Standard Model



from experiments (particle accelerators)

Particle accelerators are giant microscopes

Higher energies = smaller scales



Theory: perform calculations that predict results of events that can be seen in accelerators

Formula: Standard Model Lagrangian

$$\mathcal{L}_{SM} = -\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu -$$
$$M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) -$$
$$igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu -$$
$$W^-_\nu \partial_\nu W^+_\mu)) - \frac{1}{2} g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac{1}{2} g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2 (Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu -$$
$$Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2 (A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w (A_\mu Z^0_\nu (W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \frac{1}{2} \partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - \frac{1}{2} \partial_\mu\phi^0\partial_\mu\phi^0 -$$
$$\beta_h \left(\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac{1}{2}(H^2+\phi^0\phi^0+2\phi^+\phi^-)\right) + \frac{2M^4}{g^2}\alpha_h -$$
$$g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) -$$
$$\frac{1}{8} g^2\alpha_h \left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) -$$
$$gMW^+_\mu W^-_\mu H - \frac{1}{2} g\frac{M}{c_w^2} Z^0_\mu Z^0_\mu H -$$
$$\frac{1}{2} ig \left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) +$$
$$\frac{1}{2} g \left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \frac{1}{2} g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) +$$
$$M\left(\frac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- -$$
$$W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) -$$
$$\frac{1}{4} g^2 W^+_\mu W^-_\mu\left(H^2+(\phi^0)^2+2\phi^+\phi^-\right) - \frac{1}{8} g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-\right) -$$
$$\frac{1}{2} g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \frac{1}{2} ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2} g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) + \frac{1}{2} ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- -$$
$$g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \frac{1}{2} ig_s\,\lambda^a_{ij}(\bar q^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar e^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar\nu^\lambda(\gamma\partial+m^\lambda_\nu)\nu^\lambda - \bar u^\lambda_j(\gamma\partial +$$
$$m^\lambda_u)u^\lambda_j - \bar d^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_wA_\mu\left(-(\bar e^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar u^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar d^\lambda_j\gamma^\mu d^\lambda_j)\right) +$$
$$\frac{ig}{4c_w}Z^0_\mu\{(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar e^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda) + (\bar d^\lambda_j\gamma^\mu(\tfrac{4}{3} s_w^2 - 1 - \gamma^5)d^\lambda_j) +$$
$$(\bar u^\lambda_j\gamma^\mu(1-\tfrac{8}{3} s_w^2+\gamma^5)u^\lambda_j)\} + \frac{ig}{2\sqrt{2}}W^+_\mu\left((\bar\nu^\lambda\gamma^\mu(1+\gamma^5)U^{lep}{}_{\lambda\kappa}e^\kappa) + (\bar u^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) +$$
$$\frac{ig}{2\sqrt{2}}W^-_\mu\left((\bar e^\kappa U^{lep\dagger}{}_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar d^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar\nu^\lambda U^{lep}{}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar\nu^\lambda U^{lep}{}_{\lambda\kappa}(1+\gamma^5)e^\kappa)\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar e^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar e^\lambda U^{lep\dagger}{}_{\lambda\kappa}(1-\gamma^5)\nu^\kappa)\right) - \frac{g}{2}\frac{m^\lambda_\nu}{M}H(\bar\nu^\lambda\nu^\lambda) -$$
$$\frac{g}{2}\frac{m^\lambda_e}{M}H(\bar e^\lambda e^\lambda) + \frac{ig}{2}\frac{m^\lambda_\nu}{M}\phi^0(\bar\nu^\lambda\gamma^5\nu^\lambda) - \frac{ig}{2}\frac{m^\lambda_e}{M}\phi^0(\bar e^\lambda\gamma^5 e^\lambda) - \frac{1}{4}\,\bar\nu_\lambda\, M^R_{\lambda\kappa}\,(1-\gamma_5)\hat\nu_\kappa -$$
$$\frac{1}{4}\,\bar\nu_\lambda\, M^R_{\lambda\kappa}\,(1-\gamma_5)\hat\nu_\kappa + \frac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar u^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar u^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j)\right) +$$
$$\frac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j)\right) - \frac{g}{2}\frac{m^\lambda_u}{M}H(\bar u^\lambda_j u^\lambda_j) -$$
$$\frac{g}{2}\frac{m^\lambda_d}{M}H(\bar d^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m^\lambda_u}{M}\phi^0(\bar u^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m^\lambda_d}{M}\phi^0(\bar d^\lambda_j\gamma^5 d^\lambda_j) + \bar G^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar G^a G^b g^c_\mu +$$
$$\bar X^+(\partial^2-M^2)X^+ + \bar X^-(\partial^2-M^2)X^- + \bar X^0(\partial^2 - \frac{M^2}{c_w^2})X^0 + \bar Y\partial^2 Y + igc_wW^+_\mu(\partial_\mu\bar X^0 X^- -$$
$$\partial_\mu\bar X^+X^0) + igs_wW^+_\mu(\partial_\mu\bar Y X^- - \partial_\mu\bar X^+ Y) + igc_wW^-_\mu(\partial_\mu\bar X^- X^0 -$$
$$\partial_\mu\bar X^0 X^+) + igs_wW^-_\mu(\partial_\mu\bar X^- Y - \partial_\mu\bar Y X^+) + igc_wZ^0_\mu(\partial_\mu\bar X^+X^+ -$$
$$\partial_\mu\bar X^-X^-) + igs_wA_\mu(\partial_\mu\bar X^+X^+ -$$
$$\partial_\mu\bar X^-X^-) - \frac{1}{2} gM\left(\bar X^+X^+H + \bar X^-X^-H + \frac{1}{c_w^2}\bar X^0X^0H\right) + \frac{1-2c_w^2}{2c_w}igM\left(\bar X^+X^0\phi^+ - \bar X^-X^0\phi^-\right) +$$
$$\frac{1}{2c_w}igM\left(\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-\right) + igMs_w\left(\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-\right) +$$
$$\frac{1}{2} igM\left(\bar X^+X^+\phi^0 - \bar X^-X^-\phi^0\right) .$$

We have a formula: does it mean we understand?

The task of mathematics:

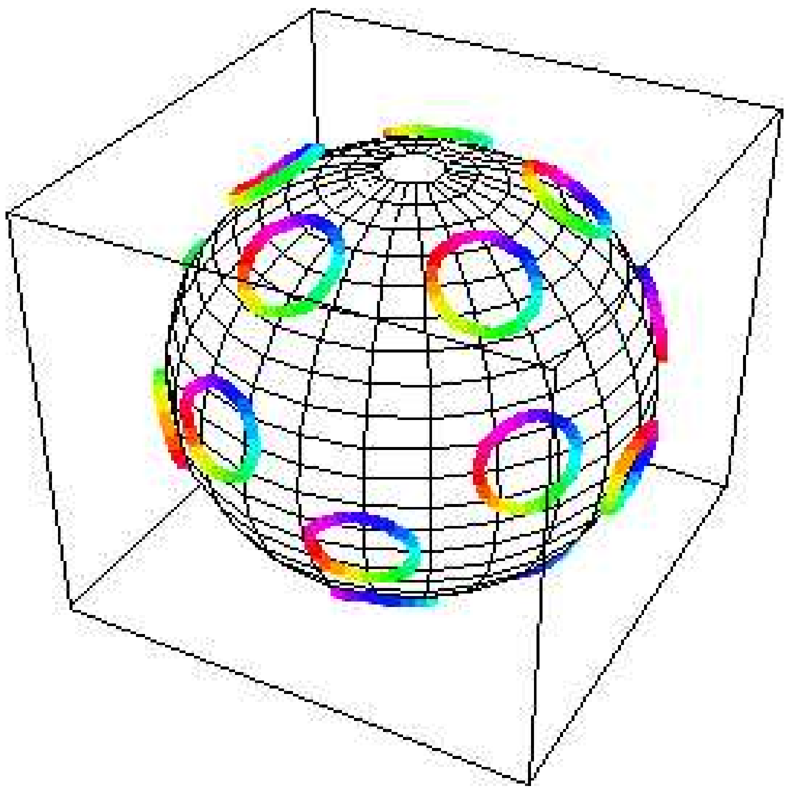
- Is there a *simple* principle behind?
- Does the formula *follow*?
- What does it *mean*?

Geometry: guiding principle for tackling complexity

“Collision II” Dawn N. Meson, San Francisco artist

Geometrization of physics

Kaluza–Klein theory



General Relativity:

gravity = metric on 4-dim spacetime

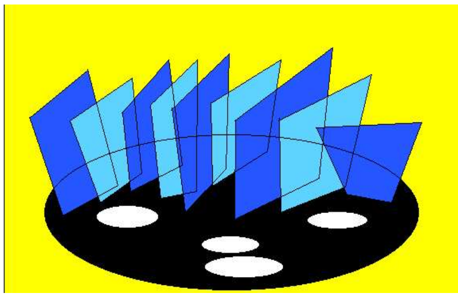
Electromagnetism: 5-dimensions

Circle bundle over spacetime

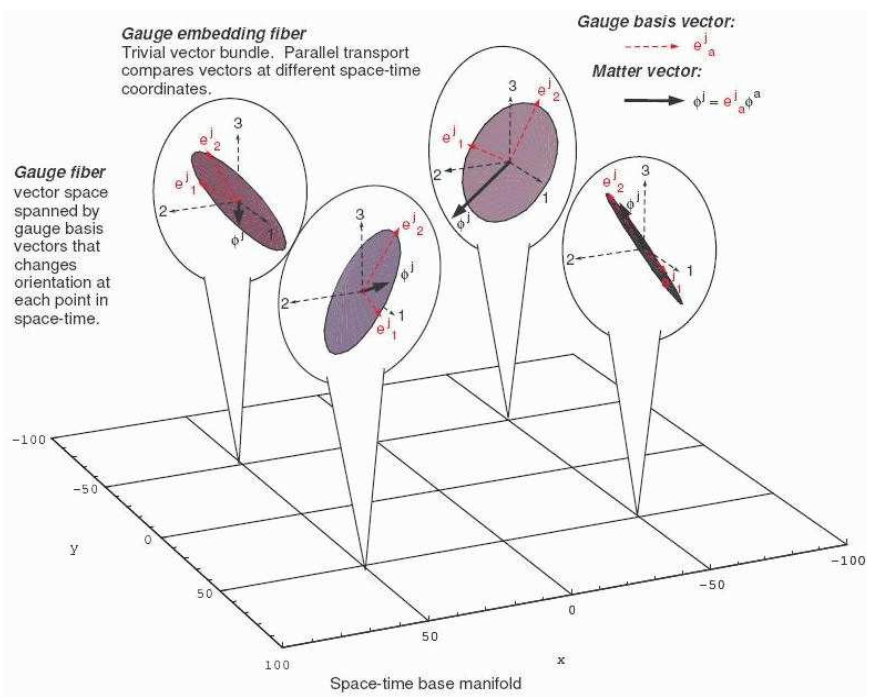
$\Rightarrow$ Gauge theories

Evolution of the Kaluza Klein idea, I

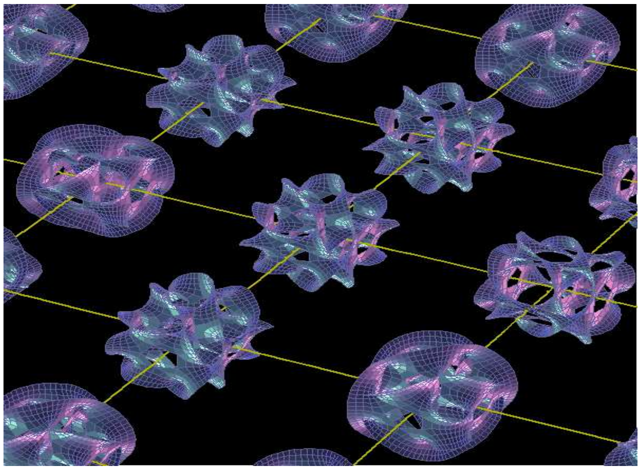
Gauge theories: vector bundles



connections and curvatures (gauge potentials, force fields)
sections (matter particles/fields)
bundle symmetries (gauge symmetries)



Evolution of the Kaluza Klein idea, II



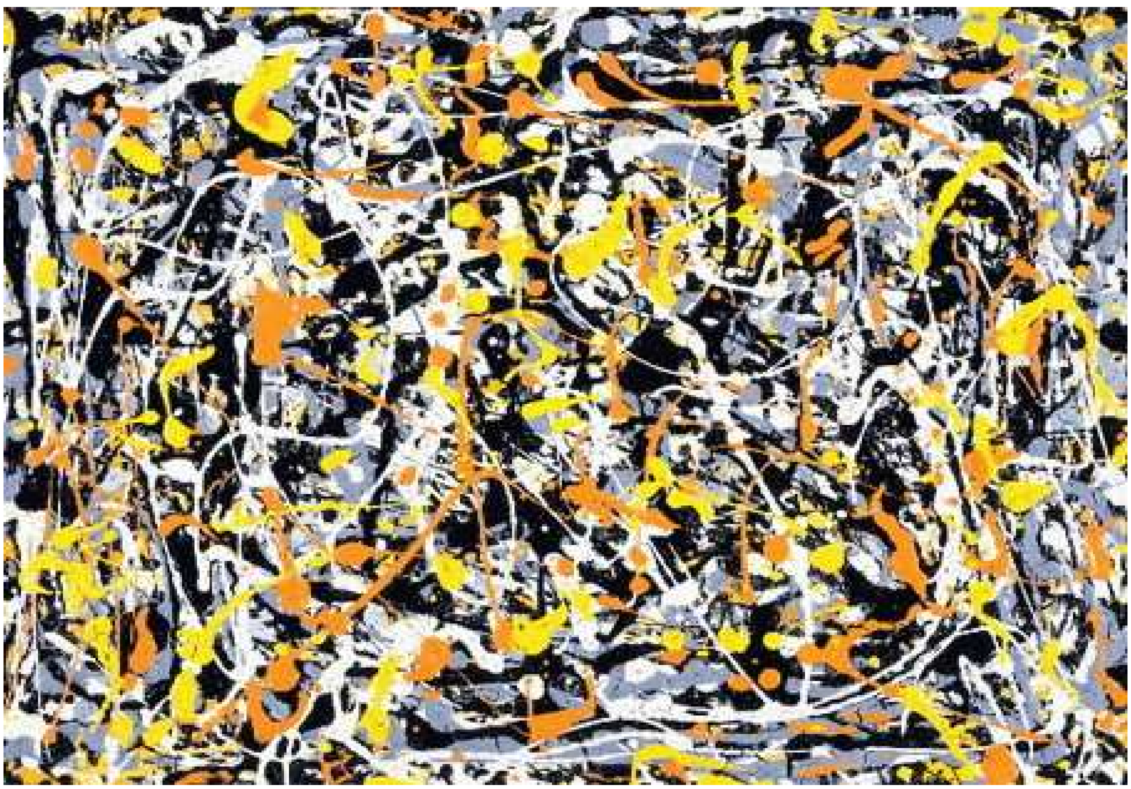
String theory: fibration of Calabi-Yau manifolds over 4-dim spacetime

"Kaluza-Klein (Invisible Architecture III)" Dawn N.Meson

Evolution of the Kaluza Klein idea, III

Noncommutative geometry (Connes, 1980s)

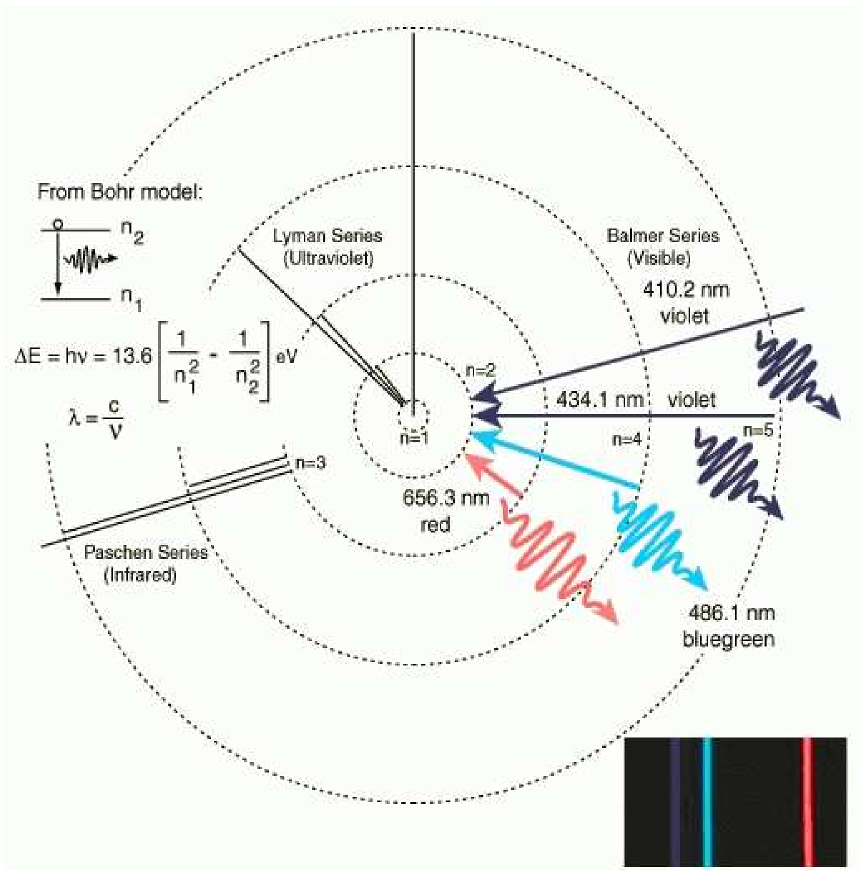
Product of spacetime by a "noncommutative space"



Jackson Pollock "Untitled N.3"

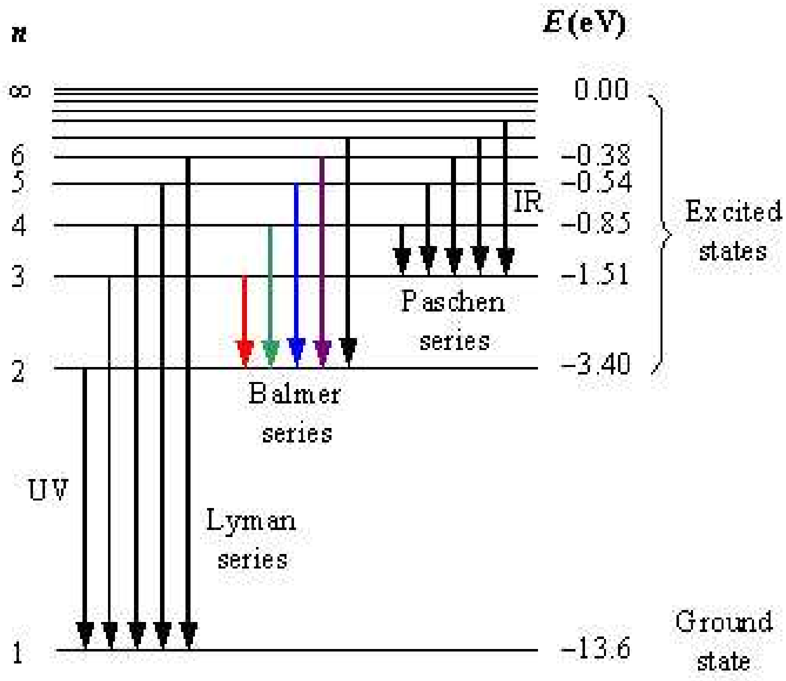
What is a noncommutative space?

Example: composition law for spectral lines



Compose when target of one is source of next:

$$G = groupoid$$



Energy levels of the hydrogen atom with some of the transitions between them that give rise to the spectral lines indicated.

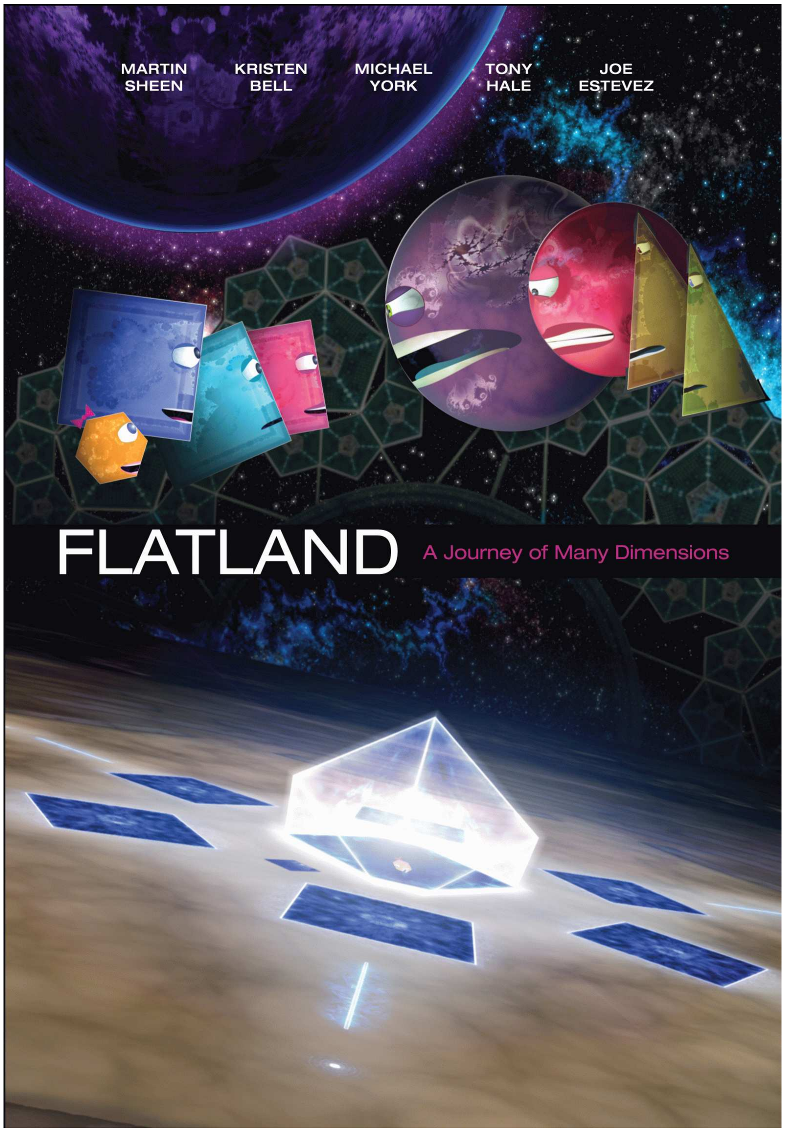
First instance of noncommutive variables
in Quantum Mechanics

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} u & v \\ x & y \end{pmatrix} = \begin{pmatrix} au + bx & av + by \\ cu + dx & cv + dy \end{pmatrix} \neq$$

$$\begin{pmatrix} au + cv & bu + dv \\ ax + cy & bx + dy \end{pmatrix} = \begin{pmatrix} u & v \\ x & y \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Observables in quantum mechanics usually don't commute $\Rightarrow$ "Uncertainty principle"
NCG: *Geometry of Quantum Mechanics*

The main idea: There are *more dimensions* than the 4-dimensions of space and time



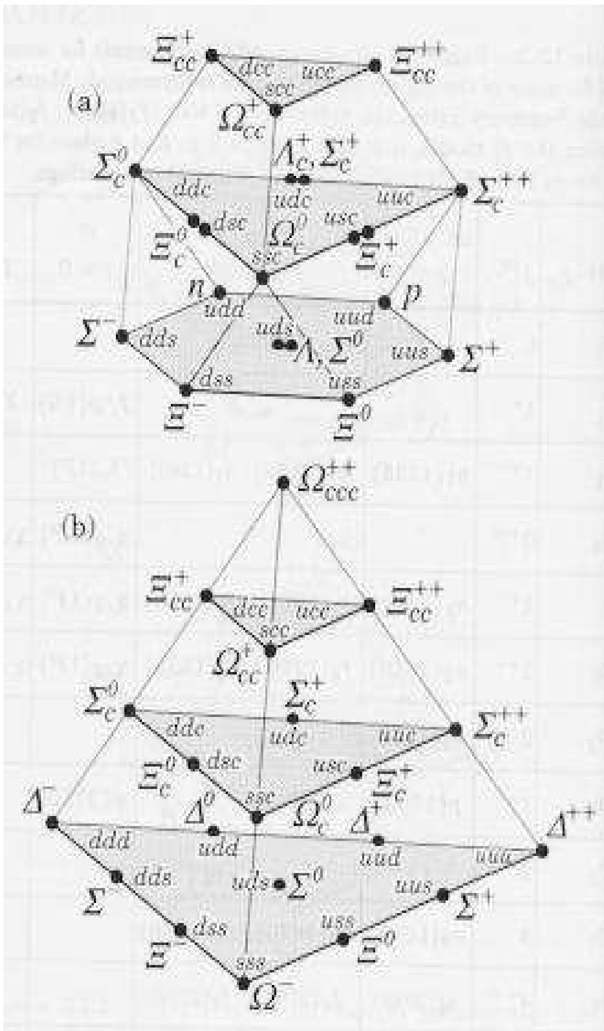
The *extra dimensions* account for forces and particles and their interactions (internal symmetries)

But when is a mathematical model a good model of the physical world?

- *Simplicity*: difficult computations follow from simple principles
- *Predictive power*: new insight on physics, new testable calculations
- *Elegance*: "entia non sunt multiplicanda praeter necessitatem" (Ockham's razor)

More than one mathematical model may be needed to explain different aspects of the same physical phenomenon

Example: Composite particles (baryons)



Classification in terms of elementary particles (quarks)

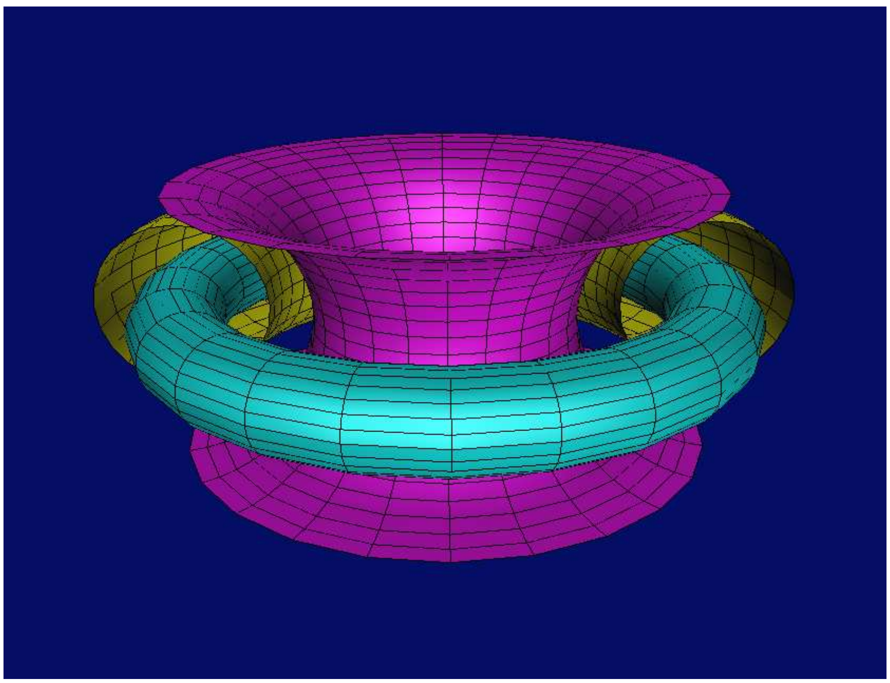
Mathematics: Lie group $SU(3)$

- Linear representations of Lie groups

Example: Noncommutative Geometry:
Standard Model Lagrangian *computed*
from simple input

Matrix algebras and quaternions

$$\mathcal{A} = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$$



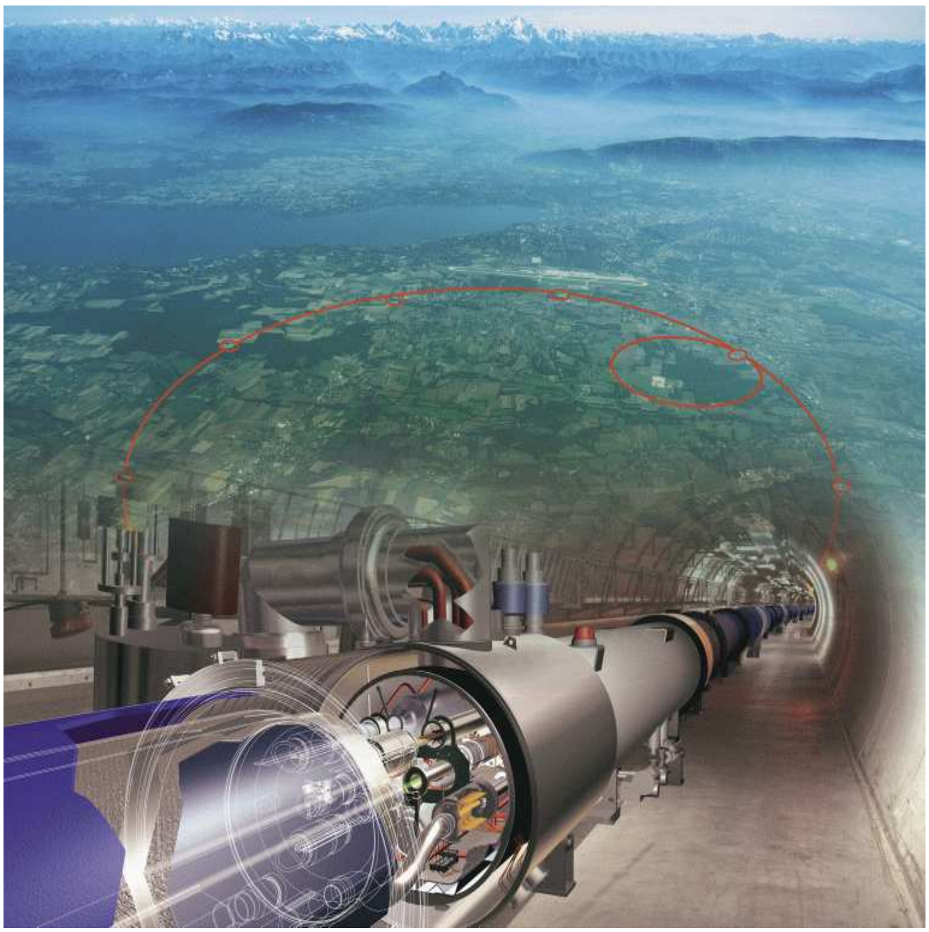
Predictions: Higgs mass, mass relation

Mathematics: Spectral triples, spectral action

Mathematics and reality

The test of experiments

(different models predict different Higgs masses)



Large Hadron Collider (CERN)

September 2008 (?)