1 One Loop Quark-Quark-Gluon Vertex Corrections

This is a detailed calculation of the one-loop vertex corrections to incoming quark anti-quark collisions. These expressions will save time in the furture when they are needed.

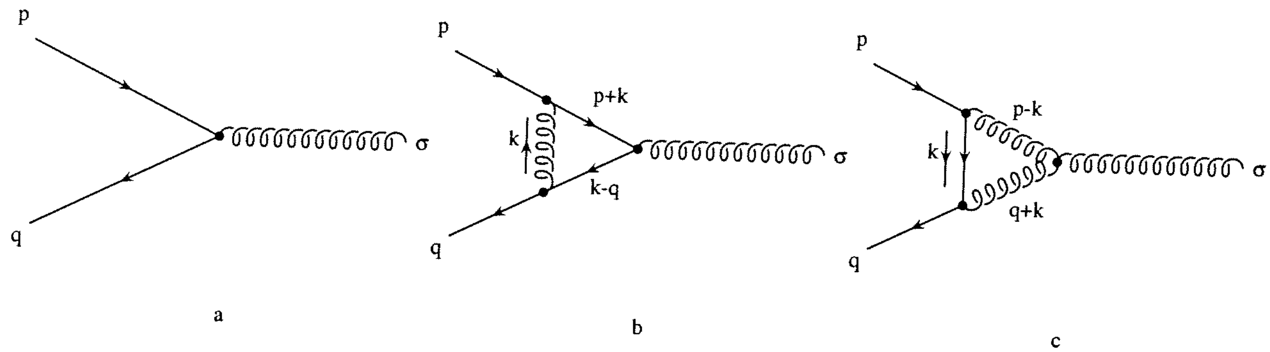

Figure 1: a) This is the standard $q\bar{q}g$ vertex. b) The first correction due to a gluon passing between the two incoming quarks. c) The second correction due to another quark passing between the two incoming quarks.

The expression (a_v^σ) for the partial diagram in Fig. 1a is

$$i\, a_v^\sigma = -i g_s T^A \bar{u}(q)\gamma^\sigma u(p). \tag{1}$$

If we define $a_{v,0}^\sigma = -g_s \bar{u}(q)\gamma^\sigma u(p)$, then we can write this as

$$i\, a_v^\sigma = i T^A a_{v,0}^\sigma. \tag{2}$$

The partial amplitude in Fig. 1b is a little more involved. We can write this expression (T_1^σ) as

$$\begin{aligned} i\, T_1^\sigma &= \bar{u}(q)\int \frac{d^n k}{(2\pi)^n}(-i g_s T^A \gamma^\mu)\frac{i(\not{k}-\not{q})}{(k-q)^2}(-i g_s T^C \gamma^\sigma)\frac{i(\not{p}+\not{k})}{(p+k)^2}(-i g_s T^A \gamma_\mu)\frac{-i}{k^2}u(p) \\ &= -g_s^3 (T^A T^C T^A)\int \frac{d^n k}{(2\pi)^n}\bar{u}(q)\frac{\gamma^\mu(\not{k}-\not{q})\gamma^\sigma(\not{p}+\not{k})\gamma_\mu}{k^2(k-q)^2(k+p)^2}u(p) \end{aligned} \tag{3}$$

The numerator can be simplified using $\{\gamma^\sigma, \gamma_\mu\} = 2\eta_\mu^\sigma$, $[\gamma^\sigma, \gamma_\mu] = 0$, and $\gamma^\mu \not{p}\gamma_\mu = 2(\epsilon - 1)\not{p}$ (we have let $d = 4 - 2\epsilon$) as

$$\begin{aligned} \gamma^\mu(\not{k}-\not{q})\gamma^\sigma(\not{p}+\not{k})\gamma_\mu &= \gamma^\mu \not{k}\gamma^\sigma\gamma_\mu \not{p} + \gamma^\mu \not{k}\gamma^\sigma\gamma_\mu \not{k} - \gamma^\mu \not{q}\gamma^\sigma\gamma_\mu \not{p} - \gamma^\mu \not{q}\gamma^\sigma\gamma_\mu \not{k} \\ &= \gamma^\mu \not{k}\gamma_\mu\gamma^\sigma \not{p} + \gamma^\mu \not{k}\gamma_\mu\gamma^\sigma \not{k} - \gamma^\mu \not{q}\gamma_\mu\gamma^\sigma \not{p} - \gamma^\mu \not{k}\gamma_\mu\gamma^\sigma \not{k} \\ &= \gamma^\sigma(2\epsilon-2)\not{k}\not{p} + \gamma^\sigma(2\epsilon-2)\not{k}\not{k} - \gamma^\sigma(2\epsilon-2)\not{q}\not{p} - \gamma^\sigma(2\epsilon-2)\not{q}\not{k} \\ &= -2\gamma^\sigma(\not{k}\not{p} + \not{k}\not{k} - \not{q}\not{p} - \not{q}\not{k}) + 2\epsilon\gamma^\sigma(\not{k}\not{p} + \not{k}\not{k} - \not{q}\not{p} - \not{q}\not{k}) \\ &= -2(\not{p}+\not{k})\gamma^\sigma(\not{k}-\not{q}) + 2\epsilon(\not{k}-\not{q})\gamma^\sigma(\not{p}+\not{k}) \end{aligned} \tag{4}$$

We also know that for massless fermions, the Dirac equation tells us $\bar{u}(q)\not{q} = 0$ and $\not{p}u(p) = 0$. This allows for the simplification of our vertex correction to be

$$i\, T_1^\sigma = -g_s^3 (T^A T^C T^A)\bar{u}(q)\int \frac{d^n k}{(2\pi)^n}\frac{\{-2(\not{p}+\not{k})\gamma^\sigma(\not{k}-\not{q}) + 2\epsilon\,\not{k}\gamma^\sigma\not{k}\}}{k^2(k-q)^2(k+p)^2}u(p) \tag{5}$$

With repeated use of the Dirac equation we simplify the following:

$$
\begin{aligned}
\not{k}\gamma^\sigma \not{k} &= -k^2\gamma^\sigma + 2k^\sigma \not{k} \\
\not{p}\gamma^\sigma \not{k} &= (-\gamma^\sigma \not{p} + 2p^\sigma) \not{k} \\
&= \gamma^\sigma(\underbrace{\not{k} \not{p}}_{0\,\text{by}\,\text{D.E.}} -2p\cdot k) + 2p^\sigma \not{k} \\
&= -\gamma^\sigma((k+p)^2 - k^2) + 2p^\sigma \not{k}
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
\not{k}\gamma^\sigma \not{q} &= \not{k}(-\not{q}\gamma^\sigma + 2q^\sigma) \\
&= 2q^\sigma \not{k} + (\underbrace{\not{q} \not{k}}_{0\,\text{by}\,\text{D.E.}} -2k\cdot q)\gamma^\sigma \\
&= 2q^\sigma \not{k} + \gamma^\sigma[(k-q)^2 - k^2]
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
\not{p}\gamma^\sigma \not{q} &= (-\gamma^\sigma \not{p} + 2p^\sigma) \not{q} \\
&= \gamma^\sigma(\underbrace{- \not{q} \not{p}}_{0\,\text{by}\,\text{D.E.}} +2p\cdot q) + \underbrace{2p^\sigma \not{q}}_{0\,\text{by}\,\text{D.E.}} \\
&= \gamma^\sigma(2p\cdot q)
\end{aligned} \tag{8}
$$

Many cancellations give

$$
\begin{aligned}
i\,T_1^\sigma = &- g_s^3(T^AT^CT^A)\bar{u}(q)\gamma^\sigma u(p)\int\frac{d^nk}{(2\pi)^n}\frac{\{2(k+p)^2 + 2(k-q)^2 - 2k^2 - 4p\cdot q - 2\epsilon k^2\}}{k^2(k-q)^2(k+p)^2} \\
&- g_s^3(T^AT^CT^A)\bar{u}(q)\int\frac{d^nk}{(2\pi)^n}\frac{\not{k}\{-4(1-\epsilon)k^\sigma - 4p^\sigma + 4q^\sigma\}}{k^2(k-q)^2(k+p)^2}u(p)
\end{aligned} \tag{9}
$$

Break $\not{k} = \gamma_\alpha k^\alpha$ and we have

$$
\begin{aligned}
i\,T_1^\sigma = &- g_s^3(T^AT^CT^A)\bar{u}(q)\gamma^\sigma u(p)\int\frac{d^nk}{(2\pi)^n}\left[\frac{2}{k^2(k-q)^2} + \frac{2}{k^2(k+p)^2} - \frac{2(1+\epsilon)}{(k-q)^2(k+p)^2} - \frac{4p\cdot q}{k^2(k-q)^2(k+p)^2}\right] \\
&- g_s^3(T^AT^CT^A)\bar{u}(q)\gamma_\alpha\int\frac{d^nk}{(2\pi)^n}\left[\frac{-4(1-\epsilon)k^\alpha k^\sigma}{k^2(k-q)^2(k+p)^2} - \frac{4k^\alpha p^\sigma}{k^2(k-q)^2(k+p)^2} + \frac{4k^\alpha q^\sigma}{k^2(k-q)^2(k+p)^2}\right]u(p) \\
= &- g_s^3(T^AT^CT^A)\bar{u}(q)\gamma^\sigma u(p)[-2(1+\epsilon)I_A + 4I_B - 4p\cdot qI_C] \\
&- g_s^3(T^AT^CT^A)\bar{u}(q)\gamma_\alpha[4q^\sigma I_D^\alpha - 4p^\sigma I_D^\alpha - 4(1-\epsilon)I_E^{\alpha\sigma}]u(p).
\end{aligned} \tag{10}
$$

And we are left to evaluate the integrals $I_A, I_B, I_C, I_D^\alpha, I_E^{\alpha\sigma}$.

$$
\begin{aligned}
I_A &\equiv \int\frac{d^nk}{(2\pi)^n}\frac{1}{(k-q)^2(k+p)^2} \\
&= \int\frac{d^nk}{(2\pi)^n}\frac{1}{(k+p+q)^2k^2} \\
&= \int_0^1 dx\int\frac{d^nk}{(2\pi)^n}\frac{1}{[k^2+(p+q)^2x(1-x)]^2} \qquad \Delta = -(p+q)^2x(1-x) \\
&= \int_0^1 dx\int\frac{d^nk}{(2\pi)^n}\frac{1}{[k^2-\Delta]^2} \qquad \int\frac{d^dk}{(2\pi)^d}\frac{1}{[k^2-\Delta]^n} = \frac{(-1)^n i}{(4\pi)^{\frac{d}{2}}}\frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)}\left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}} \quad \text{(P\&S } A.44) \\
&= \frac{i(4\pi)^\epsilon\Gamma(\epsilon)}{16\pi^2}\int_0^1 dx[-(p+q)^2x(1-x)]^{-\epsilon} \qquad n=2,\ \frac{d}{2} = 2-\epsilon \\
&= \frac{i}{16\pi^2}\left(\frac{4\pi}{-s}\right)^\epsilon\frac{\Gamma(1+\epsilon)}{\epsilon}\int_0^1 dx\,x^{-\epsilon}(1-x)^{-\epsilon} \qquad s = (p+q)^2 \\
&= \frac{i}{16\pi^2}\left(\frac{4\pi}{-s}\right)^\epsilon\frac{\Gamma(1+\epsilon)}{\epsilon}\frac{\Gamma^2(1-\epsilon)}{\Gamma(2-2\epsilon)} \qquad \int_0^1 dx\,x^{-a}(1-x)^{-b} = \frac{\Gamma(1-a)\Gamma(1-b)}{\Gamma(2-a-b)} \\
&= \frac{i}{16\pi^2}\left(\frac{4\pi}{-s}\right)^\epsilon\frac{\Gamma(1+\epsilon)}{\epsilon}\frac{1}{1-2\epsilon}\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
I_B &\equiv \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2(k \pm l)^2} \\
&= \int_0^1 dx \int \frac{d^n k}{(2\pi)^n} \frac{1}{[k^2 \pm l^2 x(1-x)]^2} \qquad \text{shift the integration} \\
&= \int_0^1 dx \int \frac{d^n \kappa}{(2\pi)^n} \frac{1}{\kappa^4} \\
&= 0
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
I_C &\equiv \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2(k-q)^2(k+p)^2} \qquad k \to k + px - qy \\
&= \int_0^1 dx \int_0^{1-x} dy \int \frac{d^n k}{(2\pi)^n} \frac{1}{[k^2 + p^2 x(1-x) + q^2 y(1-y) + 2p \cdot q x y]^3} \\
&= 2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^n k}{(2\pi)^n} \frac{1}{[k^2 + sxy]^3} \\
&= -\frac{2i}{16\pi^2} (4\pi)^\epsilon \frac{\Gamma(1+\epsilon)}{\Gamma(3)} \int_0^1 dx \int_0^{1-x} dy \left(\frac{1}{-sxy}\right)^{1+\epsilon} \\
&= \frac{i}{16\pi^2} \frac{1}{s} \left(\frac{4\pi}{-s}\right)^\epsilon \Gamma(1+\epsilon) \int_0^1 dx \int_0^{1-x} dy \left(\frac{1}{xy}\right)^{1+\epsilon} \\
&= -\frac{i}{16\pi^2} \frac{1}{s} \left(\frac{4\pi}{-s}\right)^\epsilon \frac{\Gamma(1+\epsilon)}{\epsilon} \int_0^1 dx\, x^{-1-\epsilon}(1-x)^{-\epsilon} \\
&= -\frac{i}{16\pi^2} \frac{1}{s} \left(\frac{4\pi}{-s}\right)^\epsilon \frac{\Gamma(1+\epsilon)}{\epsilon} \frac{\Gamma(-\epsilon)\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \\
&= \frac{i}{16\pi^2} \frac{1}{s} \left(\frac{4\pi}{-s}\right)^\epsilon \frac{\Gamma(1+\epsilon)}{\epsilon^2} \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
I_D^\alpha &\equiv \int \frac{d^n k}{(2\pi)^n} \frac{k^\alpha}{k^2(k-q)^2(k+p)^2} \\
&= \int \frac{d^n k}{(2\pi)^n} \int_0^1 dx \int_0^{1-x} dy \frac{(-px+qy)^\alpha}{[k^2+sxy]^3} \qquad \int d^4k \frac{\not{k}}{(k^2+\Delta)^\alpha} = 0 \\
&= 0
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
I_E^{\alpha\sigma} &\equiv \int \frac{d^n k}{(2\pi)^n} \frac{k^\alpha k^\sigma}{k^2(k-q)^2(k+p)^2} \qquad \kappa = k + px - qy \\
&= 2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^n \kappa}{(2\pi)^n} \frac{\frac{1}{n}\eta^{\alpha\sigma}\kappa^2 + (px-qy)^\alpha (px-qy)^\sigma}{[\kappa^2 + sxy]^3} \\
&= \frac{2}{n}\eta^{\alpha\sigma} \int_0^1 dx \int_0^{1-x} dy \int \frac{d^n \kappa}{(2\pi)^n} \frac{\kappa^2}{[\kappa^2+sxy]^3} \qquad \int \frac{d^d k}{(2\pi)^d} \frac{k^2}{(k^2-\Delta)^n} = \frac{(-1)^{n-1} i}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(n-\frac{d}{2}-1)}{\Gamma(n)} \left(\frac{1}{\Delta}\right)^{n-\frac{d}{2}-1} \\
&= \frac{2}{4-2\epsilon}\eta^{\alpha\sigma} \frac{i}{16\pi^2} (4\pi)^\epsilon \frac{4-2\epsilon}{2} \frac{\Gamma(\epsilon)}{\Gamma(3)} \int_0^1 dx \int_0^{1-x} dy \left(\frac{1}{-sxy}\right)^\epsilon \\
&= \frac{\eta^{\alpha\sigma}}{2} \frac{i}{16\pi^2} \frac{\Gamma(1+\epsilon)}{\epsilon} \left(\frac{4\pi}{-s}\right)^\epsilon \int_0^1 dx \int_0^{1-x} dy (xy)^{-\epsilon} \\
&= \frac{\eta^{\alpha\sigma}}{2} \frac{i}{16\pi^2} \frac{\Gamma(1+\epsilon)}{\epsilon} \left(\frac{4\pi}{-s}\right)^\epsilon \frac{1}{1-\epsilon} \int_0^1 dx (1-x)^{1-\epsilon} x^{-\epsilon} \\
&= \frac{\eta^{\alpha\sigma}}{2} \frac{i}{16\pi^2} \frac{\Gamma(1+\epsilon)}{\epsilon} \left(\frac{4\pi}{-s}\right)^\epsilon \frac{1}{1-\epsilon} \frac{\Gamma(1-\epsilon)\Gamma(2-\epsilon)}{\Gamma(3-2\epsilon)}
\end{aligned} \tag{15}
$$

So finally, if we let $4p\cdot q = 2s$ then we can simplify the result (remembering that $I_B = I_D^\alpha = 0$)

$$
\begin{aligned}
i\,T_1^\sigma &= -g_s^3(T^AT^CT^A)\bar{u}(q)\gamma^\sigma u(p)[-2(1+\epsilon)I_A + 4I_B - 4p\cdot qI_C] \\
&\quad - g_s^3(T^AT^CT^A)\bar{u}(q)\gamma_\alpha[4q^\sigma I_D^\alpha - 4p^\sigma I_D^\alpha - 4(1-\epsilon)I_E^{\alpha\sigma}]u(p) \\
&= g_s^3(T^AT^CT^A)\bar{u}(q)\gamma^\sigma u(p)[2(1+\epsilon)I_A + 4p\cdot qI_C] \\
&\quad + g_s^3(T^AT^CT^A)\bar{u}(q)\gamma_\alpha[+4(1-\epsilon)I_E^{\alpha\sigma}]u(p) \\
&= \frac{ig_s^3}{16\pi^2}\left(\frac{4\pi}{-s}\right)^\epsilon \Gamma(1+\epsilon)T^AT^CT^A\bar{u}(q)\gamma^\sigma u(p) \\
&\quad \times\left\{\frac{2(1-\epsilon)}{\epsilon(1-2\epsilon)}\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} + \frac{2}{\epsilon^2}\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} + \frac{2\epsilon(1-\epsilon)}{\epsilon(1-\epsilon)}\frac{\Gamma(1-\epsilon)\Gamma(2-\epsilon)}{\Gamma(3-2\epsilon)}\right\}
\end{aligned} \tag{16}
$$

The final expression in the large curley brackets when expanded with the prefactor $(-1)^\epsilon$ simplifies to

$$
(-1)^\epsilon\Big\{\cdots\Big\} = \frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \left(8 - \frac{4\pi^2}{3}\right) + \Big(16 - 2\pi^2 - 4\zeta(3)\Big)\epsilon + \left(32 - \frac{16\pi^2}{3} + \frac{\pi^4}{5} - 6\zeta(3)\right)\epsilon^2 + \mathcal{O}(\epsilon^3) \tag{17}
$$

Giving our final result for the first vertex correction

$$
i\,T_1^\sigma = \frac{ig_s^3}{16\pi^2}\left(\frac{4\pi}{s}\right)^\epsilon \Gamma(1+\epsilon)\,T^AT^CT^A\bar{u}(q)\gamma^\sigma u(p)\left\{\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \left(8 - \frac{4\pi^2}{3}\right) + \mathcal{O}(\epsilon)\right\} \tag{18}
$$

The partial amplitude in Fig. 1c is the second kind of vertex correction. We can write this expression (T_2^σ) with the help of the QCD three-gluon vertex $V^{\mu\nu\sigma}(p_1,p_2,p_3)$ as

$$
\begin{aligned}
i\,T_2^\sigma &= \bar{u}(q)\int\frac{d^nk}{(2\pi)^n}(-ig_sT^D\gamma_\rho)\left[\frac{-i\delta^{BD}\eta^{\nu\rho}}{(q+k)^2}\right]\left[\frac{-i\delta^{DA}\eta^{\rho\mu}}{(p-k)^2}\right]\times \\
&\qquad \Big(-g_sf_{ABC}V^{\mu\nu\sigma}(-p+k,-k-q,p+q)\Big)(-ig_sT^D\gamma_\rho)\frac{i\,\not{k}}{k^2}u(p) \\
&= -ig_s^3T^Bf_{ABC}T^A\bar{u}(q)\int\frac{d^nk}{(2\pi)^n}\frac{\gamma^\nu\,\not{k}\gamma^\mu[(q-p+2k)^\sigma\eta^{\mu\nu} + (-k-2q)^\mu\eta^{\nu\sigma} + (2p-k)^\nu\eta^{\sigma\mu}]}{k^2(k-p)^2(k+q)^2}u(p) \\
&= -ig_s^3T^Bf_{ABC}T^A\bar{u}(q)[I^{(1)} + I^{(2)} + I^{(3)}]u(p)
\end{aligned}
$$

$$
\begin{aligned}
I^{(1)} &= \int\frac{d^nk}{(2\pi)^n}\frac{\gamma^\nu\,\not{k}\gamma^\mu(q-p+2k)^\sigma\eta^{\mu\nu}}{k^2(k-p)^2(k+q)^2} \\
&= \int\frac{d^nk}{(2\pi)^n}\frac{\gamma^\nu\,\not{k}\gamma_\nu(q-p+2k)^\sigma}{k^2(k-p)^2(k+q)^2} \\
&= 2(\epsilon-1)\gamma^\alpha\int\frac{d^nk}{(2\pi)^n}\frac{k^\alpha(q-p+2k)^\sigma}{k^2(k-p)^2(k+q)^2} \\
&= 2(\epsilon-1)\gamma^\alpha\int\frac{d^nk}{(2\pi)^n}\left\{(q-p)^\sigma\frac{k^\alpha}{(\cdots)} + 2\frac{k^\alpha k^\sigma}{(\cdots)}\right\} \\
&= 2(\epsilon-1)(q-p)^\sigma\gamma^\alpha I_D^\alpha + 4(\epsilon-1)\gamma^\alpha I_E^{\alpha\sigma} \\
&= 4(\epsilon-1)\gamma^\alpha I_E^{\alpha\sigma}
\end{aligned} \tag{19}
$$

$$
\begin{aligned}
I^{(2)} &= \int\frac{d^nk}{(2\pi)^n}\frac{\gamma^\nu\,\not{k}\gamma^\mu(-k-2q)^\mu\eta^{\nu\sigma}}{k^2(k-p)^2(k+q)^2} \\
&= \gamma^\sigma\int\frac{d^nk}{(2\pi)^n}\left\{-\frac{\not{k}\,\not{k}}{(\cdots)} - \frac{2\,\not{q}\,\not{k}}{(\cdots)}\right\} \\
&= -\gamma^\sigma\gamma^\alpha\gamma^\beta I_E^{\alpha\beta} - 2\gamma^\sigma\,\not{q}I_D^\alpha \\
&= -\gamma^\sigma\gamma^\alpha\gamma^\beta I_E^{\alpha\beta}
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
I^{(3)} &= \int \frac{d^nk}{(2\pi)^n} \frac{\gamma^\nu \not{k}\gamma^\mu(2p-k)^\nu \eta^{\sigma\mu}}{k^2(k-p)^2(k+q)^2} \\
&= \int \frac{d^nk}{(2\pi)^n} \frac{(2\not{p} - \not{k})\not{k}\gamma^\sigma}{k^2(k-p)^2(k+q)^2} \\
&= \gamma^\sigma \int \frac{d^nk}{(2\pi)^n}\left\{ 2\frac{\not{p}\not{k}}{(\cdots)} - \frac{\not{k}\not{k}}{(\cdots)}\right\} \\
&= 2\gamma^\sigma \not{p} I_D^\alpha - \gamma^\sigma\gamma^\alpha\gamma^\beta I_E^{\alpha\beta} \\
&= -\gamma^\sigma\gamma^\alpha\gamma^\beta I_E^{\alpha\beta}
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
i\,T_2^\sigma &= -ig_s^3 T^B f_{ABC} T^A \bar{u}(q)\left[4(\epsilon-1)\gamma^\alpha I_E^{\alpha\sigma} - 2\gamma^\sigma\gamma^\alpha\gamma^\beta I_E^{\alpha\beta}\right]u(p) \\
&= \frac{g_s^3}{16\pi^2} T^B f_{ABC} T^A \left(\frac{4\pi}{-s}\right)^\epsilon \Gamma(1+\epsilon)\bar{u}(q)\gamma^\sigma u(p)\left\{\frac{\Gamma^2(1-\epsilon)}{\epsilon\Gamma(2-2\epsilon)}\right\} \\
&= \frac{ig_s^3}{16\pi^2} T^B [T^C, T^B]\left(\frac{4\pi}{-s}\right)^\epsilon \Gamma(1+\epsilon)\bar{u}(q)\gamma^\sigma u(p)\left\{\frac{\Gamma^2(1-\epsilon)}{\epsilon\Gamma(2-2\epsilon)}\right\} \\
&= \frac{ig_s^3}{16\pi^2} T^B [T^C, T^B]\left(\frac{4\pi}{s}\right)^\epsilon \Gamma(1+\epsilon)\,\bar{u}(q)\gamma^\sigma u(p) \times \\
&\quad \left\{\frac{1}{\epsilon} + 2 + \left(4 - \frac{2\pi^2}{3}\right)\epsilon + \left(8 - \frac{4\pi^2}{3} - 2\zeta(3)\right)\epsilon^2 + \mathcal{O}(\epsilon^3)\right\} \\
&= \frac{ig_s^3}{16\pi^2} T^B [T^C, T^B]\left(\frac{4\pi}{s}\right)^\epsilon \Gamma(1+\epsilon)\,\bar{u}(q)\gamma^\sigma u(p) \times \left\{\frac{1}{\epsilon} + 2 + \mathcal{O}(\epsilon)\right\}
\end{aligned} \tag{22}
$$

So finally we have for our two vertex corrections the following

$$
T_1^\sigma = \frac{g_s^3}{16\pi^2}\left(\frac{4\pi}{s}\right)^\epsilon \Gamma(1+\epsilon) T^A T^C T^A \bar{u}(q)\gamma^\sigma u(p) \times \left\{\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \left(8 - \frac{4\pi^2}{3}\right) + \mathcal{O}(\epsilon)\right\} \tag{23}
$$

$$
T_2^\sigma = \frac{g_s^3}{16\pi^2}\left(\frac{4\pi}{s}\right)^\epsilon \Gamma(1+\epsilon)\, T^B [T^C, T^B]\bar{u}(q)\gamma^\sigma u(p) \times \left\{\frac{1}{\epsilon} + 2 + \mathcal{O}(\epsilon)\right\} \tag{24}
$$