The Use of Gauss Integrals as Geometric Shape Descriptors for Brain Pattern Classification

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1 Introduction

We propose the use of scaled Gauss integrals, ropelength and thickness as geometric shape descriptors of curves extracted from MRI scans of human brains. These features provide a way to measure similarity based on morphology [1, 3, 4] while being orientation-independent. Curves representing the fundus of a sulcus were computed for a number of sulci in each cortical surface.

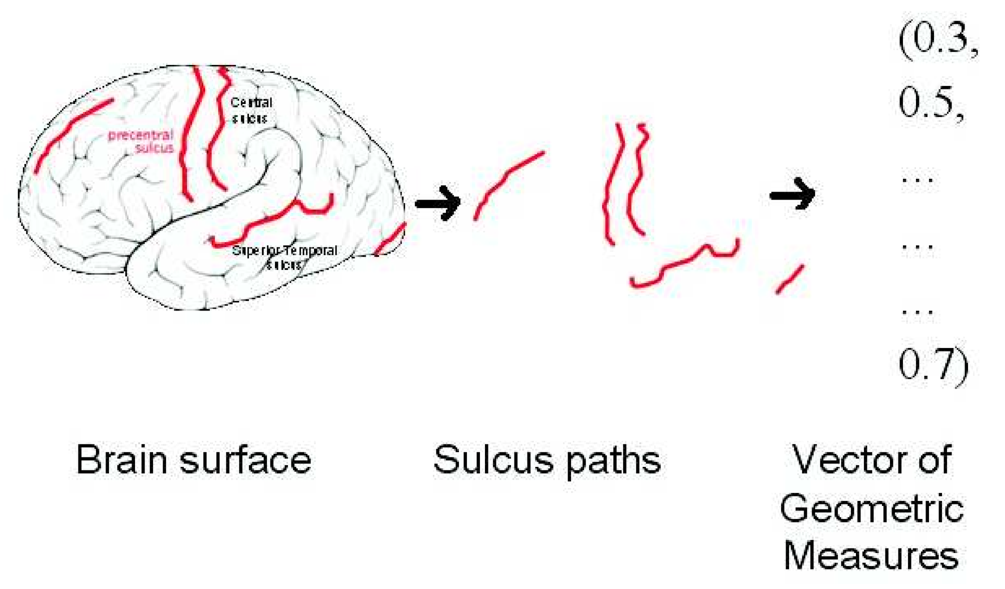

Figure 1: *Sulcus paths are space curves. A set of curve shape descriptors are then computed.*

High resolution T1-weighted, 1.5 Tesla MRI brain scans (0.86mm x 0.86mm x 1.00mm) from 15 subjects obtained from a static force experiment were used [2]. The five sulci traced on each hemisphere were the central, precentral, calcarine, superior frontal and superior temporal sulci.

2 Methods: Scaled Gauss Integrals, Curvature, Thickness and Ropelength

Consider a polygonal closed curve α. For any regular projection, each crossing can be assigned a value of $Cr(u) = \pm 1$ according to the right hand rule. The writhe of α, $Wr(\alpha)$, can be thought as average number of signed crossings of the curve, averaged over all projections $Wr(\alpha) = I_{(1,2)}(\alpha) = \Sigma_{0<i<j\leq N} W_{ij}$ as seen in figure 2.

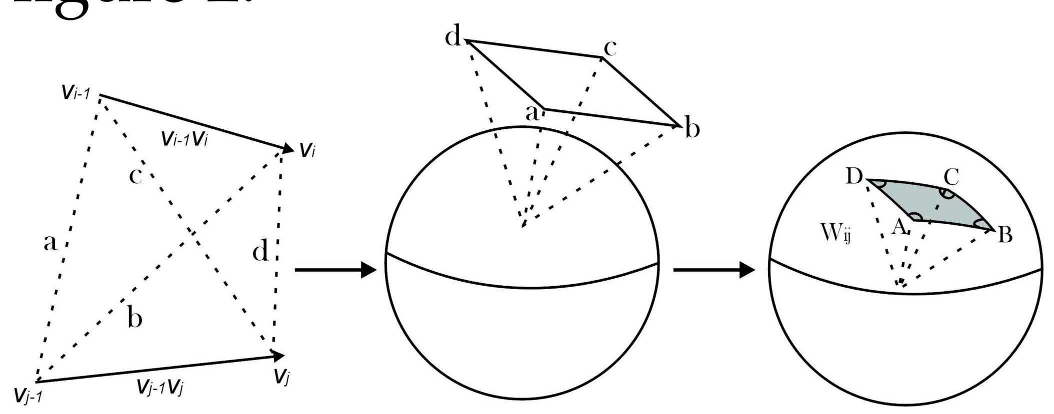

Figure 2: *The writhe contribution W_{ij} obtained from a pair of edges i and j.*

where W_{ij} is the contribution to writhe of the line segments i and j of α. W_{ij} can be computed as

$$W_{ij} = \frac{1}{2\pi}\int_{s_{i-1}}^{s_i}\int_{s_{j-1}}^{s_j} w(t_1, t_2)dt_1dt_2$$

where

$$w(t_1, t_2) = \frac{[\alpha'(t_1), \alpha(t_1) - \alpha(t_2), \alpha'(t_2)]}{|\alpha(t_1) - \alpha(t_2)|^3}$$

Another measure for curves is the *average crossing number* which is defined by taking the absolute value of the integrand $I_{|1,2|}(\alpha) = \Sigma_{0<i<j\leq N} |W_{ij}|$ where $w(t_1, t_2)$ and W_{ij} were defined previously. By constructing various combinations of W_{ij}, we can create a whole set of structural measures. The features were calculated with the Bio-Structural Classification Database, available at www.bioclassification.org

3 Results

Figure 3: *MDA projection shows a clear differentiation between male, female, left and right hemisphere.*

It was possible to differentiate sulcal paths from the left and right hemispheres, and also between males and females for every sulcus studied.

4 Conclusions

We developed a classification protocol for discrimination of sulcal curves extracted from MRI scans of human brains. The associated high dimensional feature vectors were based on a family of geometric measures involving combinations of writhe, average crossing number, ropelength and thickness. In our preliminary results, an automatic differentiation between sulcal paths from the left or right hemispheres, and male vs. female classification were achieved.

Acknowledgments

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