Quantitative Evaluation of Three Cortical Surface Flattening Methods

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Abstract

During the past decade several computational approaches have been proposed for mapping highly convoluted surfaces of the human brain to simpler geometric objects such as a sphere or a plane. We report the results of a quantitative comparison of FreeSurfer, CirclePack, and LSCM with respect to measurements of geometric distortion and computational speed. Our results indicate that FreeSurfer performs best with respect to a global measurement of metric distortion whereas LSCM is computationally much more efficient and performs both FreeSurfer and CirclePack with respect to angular distortion and a local measurement of metric distortion.

Methods

Let $K$ be a simple-connected triangulated closed surface \( \{v_k, f_k\} \subset \{v_k, f_k\} \) where \( v_k \) is a set of vertices with \( 2 \geq n \) and $T$ is a set of $m$ triangles upon which feet of vertices are $v_k$ and $v_k$. Assume that $H$ is linear on each triangle $T$ and $\mathbf{d}_{ij}$ and $\mathbf{d}_{jk}$ the geodesic distances between the vertices $v_i$ and $v_j$. Denote by $T_{ij}$ the oriented area of the triangle $T$ and by $T_{ik} = \mathbf{d}_{ij} \times \mathbf{d}_{jk}$ the geometric mean of the triangle $T_i$.

FreeSurfer

FreeSurfer [1] is a popular software package for cortical surface flattening that exploits minimization of the metric distortion of the flattened cortical surface. Define the mean-squared energy functions related to metric energy and oriented area respectively as:

\[
J_1 = \frac{1}{2m} \sum_{i=1}^{m} \left( T_{ij} \right)^2, \quad J_2 = \frac{1}{2m} \sum_{i=1}^{m} P \left( A(T_i)^2 \right) \left( A(T_i)^2 \right)
\]

where $T_{ij}$ denotes the set of vertices which are geodesically neighbors of the vertex $i$ and $P \left( A(T_i)^2 \right) \left( A(T_i)^2 \right)$ is the penalty function for the oriented area $A(T_i)$. Then, the complete functional becomes

\[
J = J_1 + J_2
\]

where $J_1$ and $J_2$ reflect the relative importance of unfolding versus the minimization of metric distortion.

CirclePack

CirclePack [2] is a quasi-conformal flattening method and depends solely on the maximization of the oriented area of the cortical surface. It can be described as follows: a collection of circles $C_k$ is a circle packing in the plane, one circle for each vertex $v_k$, with the property that $C(v_k)$ and $C(v_j)$ are largest whenever $v_k$ and $v_j$ are adjacent. The Circle Packing Theorem states that given any disk graph $G$ and any assignment of positive number $\lambda_k, \lambda_j \in R^+$ to the whole boundary vertices $v_k, v_j, \ldots, v_n$, there is a unique circle packing (up to Euclidean isometry) in the plane having boundary circles $C(v_k)$ being the radii $\lambda_k$.

We are to solve an optimization problem of finding the metric distortion of the brain using the circle packing method.

LSCM

LSCM [3] is a conformal flattening method. Suppose that $K$ is a topological disk. When restricting $H$ on one of the triangles of $T$, the Converse-Heinemann equation states that $H$ is conformal on $T$ if and only if the following equality holds true for all $T$: $\frac{1}{2 \pi} \sum_{i=1}^{n} \phi_i = 0$. This conformal criterion generally cannot be completely satisfied on the whole $K$, so minimization of violation of this condition is used to construct the quasi-conformal in the least square sense:

\[
\min C(K) = \frac{1}{2 \pi} \sum_{i=1}^{n} \phi_i
\]

where $\phi_i$ are the weights assigned to the triangle $T_i$. The metric distortion is then computed using the normalized inner product.

Angular Distortion

Angular distortion is defined to reflect the difference between corresponding angles of the cortical surface $K$ and the flat map $\{\mathbf{v}_k, f_k\}$. For the oriented area $A(T_i)$ we have $A(T_i) = \left| \mathbf{d}_{ij} \times \mathbf{d}_{jk} \right|$. All angles on the cortical surface are normal using the so-called “market share” of angles at vertex $v_i$.

\[
\frac{\sum_{k} \left| \mathbf{d}_{ik} \times \mathbf{d}_{jk} \right|}{\sum_{k} \left| \mathbf{d}_{ij} \times \mathbf{d}_{jk} \right|}
\]

where $T_{ij}$ denotes the triangle formed by the vertices $v_i, v_j, v_k$. The angle $\angle(v_i, v_k)$ on $K$ and $\mathbf{d}_{ij}$, $\mathbf{d}_{jk}$ denote the angle $\angle(v_i, v_k)$ on $K$ and $\mathbf{d}_{ij}$, $\mathbf{d}_{jk}$ denote the angle $\angle(v_i, v_k)$ on $K$.

Metric Distortion

The sum of metric distortion reflecting the global information (metric distortion) $D$ is measured as follows:

\[
D = \frac{1}{2m} \sum_{i=1}^{m} P \left( A(T_i)^2 \right) \left( A(T_i)^2 \right)
\]

where $\theta_i$ denotes the determined intensities of neighbor vertices of the vertex $v_k$. Then, $\theta_i = \frac{1}{2m} \sum_{i=1}^{m} P \left( A(T_i)^2 \right) \left( A(T_i)^2 \right)$. Then, the complete functional becomes

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\]

Here we move the minimization process inside the first summation, $\theta_i$, the minimization process is done independently on its subgraph for each vertex $v_k$.

\[
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Results

Cerebral hemispheres

Left cerebral hemispheres were extracted from high-resolution T1-weighted MR brain volumes obtained from the Montreal Neurological Institute (MNI) and the University of Pennsylvania (UPENN).

Cerebral surfaces

The left cerebral hemisphere of a human subject was extracted using FreeSurfer software. CirclePack and LSCM were applied to the surface generated by FreeSurfer to compare the differences in the curvature and metric distortion between the two methods. The results showed that LSCM produced a more accurate representation of the cerebral surface compared to the other two methods.

Conclusions

- LSCM preserved local angular information during flattening whereas FreeSurfer did not perform as well as expected due to the fact that the triangle of the cortical mesh was not equivalent.
- For all cerebrospinal spaces, FreeSurfer performed both conformal methods with respect to the preservation of metric information, however for the cerebral hemispheres, LSCM performed nearly as well as FreeSurfer and was clearly superior to FreeSurfer and CirclePack with regard to the preservation of angular information, metric information and computational efficiency.
- By preserving angular information and adequately preserving metric information, conformal methods such as LSCM may offer advantages for some neuroscience applications over methods such as FreeSurfer, which preserves only metric information and is computationally inefficient.

References