MAC 2313, Section 04 with Dr. Hurdal
Spring 2008 – Test 2

Name: _____

As stated in class, you are allowed to bring to the test one 8.5x11 inch page, written on both sides. Calculators are allowed. Notebooks and textbooks are NOT allowed. There will be 10 marks allocated for clear and well written mathematics solutions. This test will be graded out of 100.

1. (10 marks) A metal box with dimensions 1 m by 1.5 m by 2 m is to be covered with gold leaf that is 0.001 mm thick. Use differentials to approximate the volume of gold leaf needed to cover the box. Note: be careful with your units!

2. (15 marks) Given the surface $f(x, y) = x^2 - y^2$,
(a) Sketch at least 4 level curves.
(b) Find a linear approximation of $f(x, y)$ at $(2, 1)$.
(c) Use your result from (b) to estimate $f(2.10, 0.99)$.

3. (20 marks) Given the curve $\mathbf{r}(t) =< e^t, e^t \sin t, e^t \cos t >$ and the point $P = (1, 0, 1)$:
(a) Find the unit tangent, unit normal and binormal vectors of $\mathbf{r}(t)$ at P.
(b) Find the equation of the osculating plane of $\mathbf{r}(t)$ at P.

4. (15 marks) (a) Sketch the curve $\mathbf{r}(t) =< 1 + t, t^2, 3 >$, and be sure to indicate the direction of motion.
(b) Sketch $\mathbf{r}(t)$, $\mathbf{v}(t)$ and $\mathbf{a}(t)$ at $t = 1$.
(c) Find the vector equation of the tangent line to the curve at $t = 1$.
(d) What is the curvature at $t = 1$?

5. (10 marks) A projectile is fired from a position of 200 m above ground with an initial speed of 250 m/s and angle of elevation of 25°. Find the range of the projectile and maximum height reached.

6. (10 marks) The wave heights h (in feet) on the open sea depend on the speed of the wind v (in knots) and the length of time t (in hours) that the wind has been blowing. Values of the function $h = f(v, t)$ are given below:

		Duration in hours (t)		
		20	30	40
Wind Speed in knots (v)	40	17	18	19
	50	28	31	33
	60	40	45	48

(a) Estimate the value of $f_t(50, 30)$ by averaging the values of the partial derivatives taken from either side.
(b) Interpret the meaning of your answer from part (a).

7. (10 marks) Given the function $f(x,y) = \dfrac{x}{x^2+y^2}$:

(a) Evaluate $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.

(b) If $x(s,t) = s\sin t$ and $y(s,t) = \sqrt{s^2+t^2}$, evaluate $\dfrac{\partial f}{\partial s}$.

Bonus (10 marks): The ideal gas law states that for a gas confined to a container with volume V cm^3 and temperature T° Kelvin, the pressure P exerted on the container wall is given by $P = RT/V$ where R is a constant. Suppose that that an ideal gas is in a vessel with volume 1000 cm^3 at 600 K. Which will have a bigger effect on pressure: a small decrease in volume or a small increase in temperature? Show your reasoning.