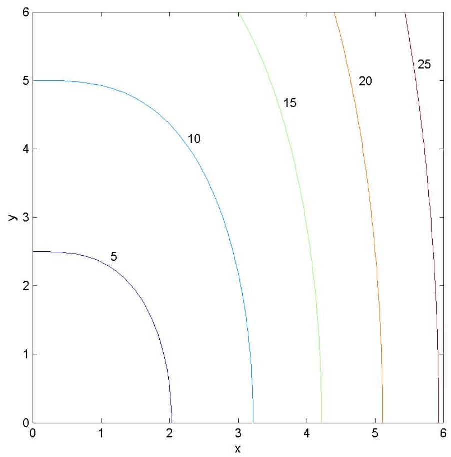
MAC 2313, Section 04 with Dr. Hurdal
Spring 2008 – Test 3

Name: ________________________

As stated in class, you are allowed to bring to the test one 8.5x11 inch page, written on both sides. Calculators are allowed. Notebooks and textbooks are NOT allowed. There will be 10 marks allocated for clear and well written mathematical solutions. This test will be graded out of 100.

1. Given the function $f(x,y) = 4 + x^3 + y^3 - 3xy$:
a) (5 marks) What is the rate of change of f at $(0,1)$ in the direction of $< 1, 2 >$?
b) (2 marks) In what direction is the maximum rate of change of f at $(0,1)$?
c) (10 marks) Find and classify all the critical points of $f(x,y)$.

2. The contour map below shows the temperature T, in degrees Celsius of region $R = [0, 6] \times [0, 6]$ where x and y are given in thousands of kilometers.
a) (3 marks) Estimate the of the rate of change of the temperature T at (3,3) in the direction towards the point $(2, 1)$ (i.e. estimate the directional derivative of T).
b) (10 marks) Use the midpoint rule with $m = 3$ and $n = 2$ to estimate the average temperature of R.



3. (7 marks) Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for $ye^x + 2ze^{x+y+z} = e^z$.

4. (15 marks) The base of an open aquarium is made with slate and the sides are made of glass. The slate costs five times as much (per unit area) as glass and the volume of the aquarium is to be 20 million cm^3. Use Lagrange multipliers to find the dimensions of the aquarium that minimize the cost of the materials.

5. (13 marks) Find the mass of the metal plate bounded by the curves $y = x^2 + 1$ and $y = x + 3$ where the density of the plate is given by $\rho(x, y) = x$ kg/m^2.

6. (10 marks) Evaluate $\int_{-\sqrt{\pi/2}}^{0} \int_{x^2}^{\pi/2} x^3 \sin(y^3)\, dy\, dx$ by reversing the order of integration.

7. (15 marks) Find the volume of the solid in the first octant that is bounded on the top and bottom by the paraboloids $z = 36 - x^2 - y^2$ and $z = 8x^2 + 8y^2$ and bounded on the sides by the xz-plane and the yz-plane.

Bonus (10 marks): The maximum and minimum values of a function $f(x, y, z)$ subject to two constraints of the form $g(x, y, z) = k$ and $h(x, y, z) = c$ (where k and c are constants) can be determined by solving the system of equations given by

$$\bigtriangledown f(x, y, z) = \lambda \bigtriangledown g(x, y, z) + \mu \bigtriangledown h(x, y, z), \qquad g(x, y, z) = k \qquad h(x, y, z) = c$$

where λ and μ are Lagrange multipliers. Use this approach to solve the following problem.

The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on the ellipse that are nearest to and farthest from the origin.
Hint: The plane is one constraint function, the paraboloid is your second constraint function and you want to optimize the distance of the point (x, y, z) to the origin.