MAC 2313, Section 04 with Dr. Hurdal
Spring 2008 – Test 4

Name: ________________________

As stated in class, you are allowed to bring to the test one 8.5x11 inch page, written on both sides. Calculators are allowed. Notebooks and textbooks are NOT allowed.

1. (15 marks) a) Show that the vector field of $\mathbf{F} =< 2x, 2y >$ is conservative.
b) Sketch this vector field.
c) On your sketch, draw a curve C_1 in the 2nd quadrant such that $\int_{C_1} \mathbf{F} \cdot d\mathbf{r} < 0$. Explain your answer.
d) If C_2 is the counterclockwise-oriented circle with radius 1 and centered at $(1, 1)$, is $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$ positive, negative or zero? Explain.

2. (10 marks) Find the area of the surface $x+2y+z=4$ that lies inside the cylinder $x^2+y^2=4$ in the first octant.

3. (15 marks) Evaluate $\displaystyle\int\int_R \frac{2x-y}{2}\, dA$ where R is the region bounded by the lines $y = 2x$, $y = -2x+8$, $y = 2x-2$, $y = -2x+4$. Use the transformation $x = \dfrac{u+v}{2}$, $y = u - v$.

4. (10 marks) a) Find the gradient vector field of $f(x, y) = (x + y + 2xy)^2$.
b) If $\mathbf{F} = \nabla f$, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is curve along the ellipse $x^2 + 4y^2 = 4$ from $(2, 0)$ to $(0, 1)$ in a counterclockwise direction.

5. (10 marks) Find the volume of the region bounded above by the sphere $x^2 + y^2 + z^2 = 2$ and below by the paraboloid $z = x^2 + y^2$.

6. (15 marks) Find the work done by the force field $\mathbf{F} =< -yz, xz, xy >$ on a particle that moves along the line segment from $(0, -1, 1)$ to $(-2, 0, 3)$ and then moves counterclockwise along the bottom half of the circle $x^2 + y^2 = 4$ in the plane $z = 3$ from $(-2, 0, 3)$ to $(2, 0, 3)$.

Bonus (7 marks): Show that if the vector field $\mathbf{F} =< P, Q, R >$ is conservative and P, Q, R have continuous first-order partial derivatives, then

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \qquad \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x} \qquad \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}.$$