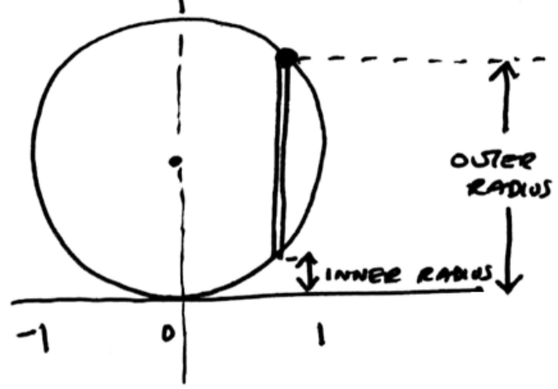


BY "WASHER" METHOD :

$$x^2+(y-1)^2 = 1^2$$

$$\Rightarrow \quad y-1 = \pm\sqrt{1-x^2}$$

$$\Rightarrow \quad y = 1+\sqrt{1-x^2} \quad \text{(OUTER RADIUS)}$$

$$\text{OR} \quad y = 1-\sqrt{1-x^2} \quad \text{(INNER RADIUS)}$$

$$\text{So } \delta V \approx \pi\left\{ OR^2 - IR^2 \right\}\delta x$$

$$= \pi\left\{ \left(1+\sqrt{1-x^2}\right)^2 - \left(1-\sqrt{1-x^2}\right)^2 \right\}\delta x$$

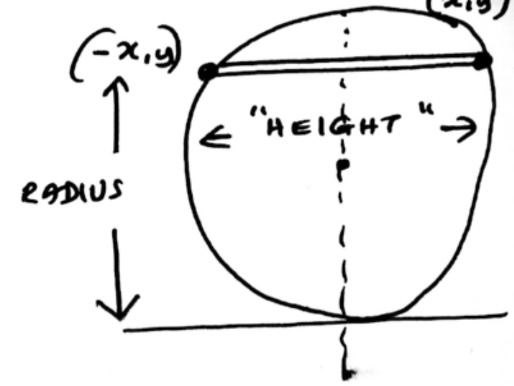
$$= 4\pi\sqrt{1-x^2}\,\delta x$$

$$\Rightarrow \quad V = 4\pi\int_{-1}^{1}\sqrt{1-x^2}\,dx \quad \text{EVEN}$$

$$= 8\pi\int_0^1\sqrt{1-x^2}\,dx$$

$= 8\pi$ area of quarter circle of radius 1

$$= 8\pi\,\frac{1}{4}\,\pi\,1^2 = 2\pi^2$$



BY "SHELL" METHOD

$$x^2+(y-1)^2 = 1 \quad \Rightarrow$$

$$x = \pm\sqrt{1-(y-1)^2}$$

$$\Rightarrow \quad \text{"HEIGHT"} = 2\sqrt{1-(y-1)^2}$$

$$= 2\sqrt{2y-y^2}$$

$$\text{So } \delta V \approx 2\pi \text{ RADIUS "HEIGHT" THICKNESS}$$

$$= 2\pi\,y\,2\sqrt{2y-y^2}\;\delta y$$

$$= 4\pi\,y\sqrt{2y-y^2}\;\delta y$$

$$\Rightarrow \quad V = \int_{y=0}^{y=2} 4\pi y\sqrt{2y-y^2}\,dy$$

Put $y = 1+\sin(u) \Rightarrow$

$$\frac{dy}{du} = 0+\cos(u) \quad \text{and}$$

$$\sqrt{2y-y^2} = \sqrt{1-\sin^2 u} = \cos(u). \text{ Then}$$

$$V = \int_{y=0}^{y=2} 4\pi y\sqrt{2y-y^2}\,dy =$$

$$\int_{u=\arcsin(0-1)}^{u=\arcsin(2-1)} 4\pi y\sqrt{2y-y^2}\,\frac{dy}{du}\,du$$

$$= \int_{-\pi/2}^{\pi/2} 4\pi\left(1+\sin(u)\right)\cos^2 u\,du$$

$$= 2\pi\int_{-\pi/2}^{\pi/2} 2\cos^2 u\,du + \underbrace{\int_{-\pi/2}^{\pi/2} 4\pi\sin(u)\cos^2(u)\,du}_{\text{ODD}}$$

$$= 2\pi\int_{-\pi/2}^{\pi/2}\left\{1+\cos(2u)\right\}du + 0$$

EVEN

GO HERE

$$= 4\pi\int_0^{\pi/2}\left\{1+\cos(2u)\right\}du$$

$$= 4\pi\left\{u+\frac{1}{2}\sin(2u)\right\}_0^{\pi/2}$$

$$= 4\pi\left\{\frac{\pi}{2}+\frac{1}{2}\sin(\pi) - 0 - \frac{1}{2}\sin(0)\right\}$$

$$= 2\pi^2$$