Put $L = \lim_{x \to \infty} x\left(e^{1/x} - 1\right)$

$= \lim_{x \to \infty} \dfrac{e^{1/x} - 1}{1/x}$, which is of type $\frac{0}{0}$

Then L'Hôpital's rule yields

$$L = \lim_{x \to \infty} \frac{\frac{d}{dx}\left(e^{1/x} - 1\right)}{\frac{d}{dx}\left(1/x\right)}$$

$$= \lim_{x \to \infty} \frac{e^{1/x} \cdot \left(\frac{-1}{x^2}\right) - 0}{-1/x^2} = \lim_{x \to \infty} e^{1/x} = e^0 = 1$$

Alternatively, using the substitution $u = 1/x$,

$$L = \lim_{u \to 0^+} \frac{e^u - 1}{u}$$

$$= \lim_{u \to 0^+} \frac{1 + u + u\,O[u] - 1}{u}$$

$$= \lim_{u \to 0^+} 1 + O[u] = 1 + 0 = 1$$

SEE FIRST PAGE OF SUPPLEMENTARY MATERIALS FOR MONDAY, MAY 25