By the product rule, $y = f(x) = x(5-x)^{1/2}$

$$\Rightarrow \quad \frac{dy}{dx} = f'(x) = \frac{d}{dx}(x)(5-x)^{1/2} + x\frac{d}{dx}\left[(5-x)^{1/2}\right]$$

$$= 1\cdot(5-x)^{1/2} + x\left\{\frac{1}{2}(5-x)^{-1/2}(0-1)\right\}$$

Use chain rule: $u = 5-x$, $z = u^{1/2} \Rightarrow$

$$\frac{d}{dx}\left((5-x)^{1/2}\right) = \frac{dz}{dx} = \frac{dz}{du}\frac{du}{dx} = \frac{1}{2}u^{-1/2}(0-1) = -\frac{1}{2}(5-x)^{-1/2}$$

$$= (5-x)^{1/2} - \frac{1}{2}x(5-x)^{-1/2}$$

$$= \frac{2\cdot(5-x)}{2\sqrt{5-x}} - \frac{x}{2\sqrt{5-x}}$$

$$= \frac{10-3x}{2\sqrt{5-x}}$$

So $m_1 = \left.\frac{dy}{dx}\right|_{x=1} = f'(1) = \frac{10-3}{2\sqrt{5-1}} = \frac{7}{4}$

and $m_2 = \left.\frac{dy}{dx}\right|_{x=4} = f'(4) = \frac{10-12}{2\sqrt{5-4}} = -1$

$\therefore$ Tangent line at $(a_1, b_1) = (1, 2)$ is

$$y - b_1 = m_1(x - a_1)$$

$$\Rightarrow \quad y - 2 = \frac{7}{4}(x-1)$$

$$\Rightarrow \quad y = \frac{7}{4}x + \frac{1}{4}$$

and tangent line at $(a_2, b_2) = (4, 4)$ is

$$y - b_2 = m_2(x - a_2)$$

$$\Rightarrow \quad y - 4 = (-1)(x-4)$$

$$\Rightarrow \quad y = -x + 8$$

or $\quad x + y = 8$