

$$x = 2y^2 \Rightarrow y^2 = \frac{x}{2} \Rightarrow y = \pm \sqrt{\frac{x}{2}} = \pm \frac{\sqrt{x}}{\sqrt{2}}$$

Line meets parabola
where both $x = 1 - y$
and $x = 2y^2 \Rightarrow$

$$1 - y = 2y^2 \Rightarrow 2y^2 + y - 1 = 0$$

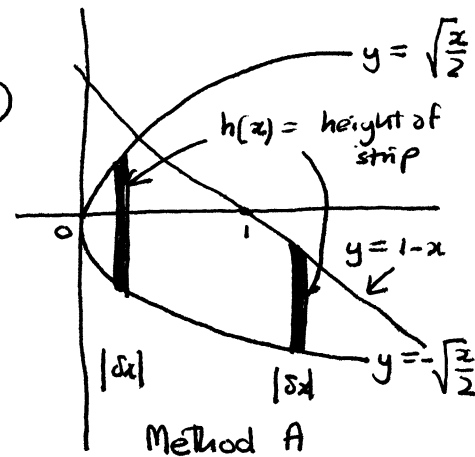
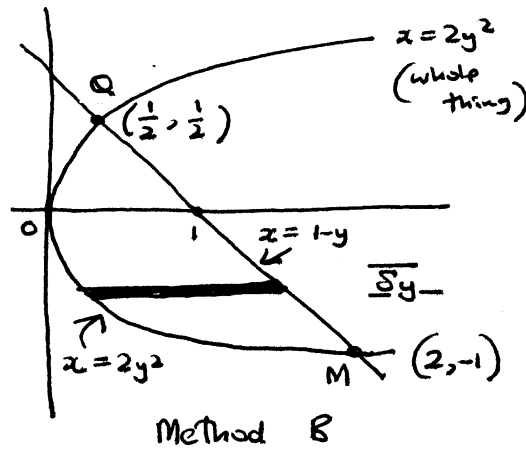
$$\Rightarrow (2y - 1)(y + 1) = 0$$

$$\Rightarrow y = \frac{1}{2} \text{ or } y = -1$$

$$\Rightarrow x = 2\left(\frac{1}{2}\right)^2 \text{ or } 2(-1)^2,$$

$$\text{i.e., } \frac{1}{2} \text{ or } 2$$

Hence Q is $\left(\frac{1}{2}, \frac{1}{2}\right)$ and M is $(2, -1)$ (and the lack of scale is lousy, but who cares about that?)



Method B (easier): $\delta A = \{(1 - y) - (2y^2)\} \delta y + o(\delta y)$ for $-1 \leq y \leq \frac{1}{2}$

$$\Rightarrow A = \int_{-1}^{\frac{1}{2}} (1 - y - 2y^2) dy = \left(y - \frac{1}{2}y^2 - \frac{2}{3}y^3 \right) \Big|_{-1}^{\frac{1}{2}} =$$

$$\frac{1}{2} - \left(\frac{1}{2}\right)^3 - \frac{2}{3}\left(\frac{1}{2}\right)^3 - \left(-1 - \frac{1}{2}(-1)^2 - \frac{2}{3}(-1)^3\right) = \frac{1}{2} - \frac{1}{8} - \frac{1}{12} + 1 + \frac{1}{2} - \frac{2}{3} = \frac{9}{8}$$

Method A: $\delta A = h(x) \delta x + o(\delta x)$ for $0 \leq x \leq 2$ where

$$h(x) = \begin{cases} \sqrt{\frac{x}{2}} - (-\sqrt{\frac{x}{2}}) = \frac{1}{\sqrt{2}}\sqrt{x} + \frac{1}{\sqrt{2}}\sqrt{x} = \frac{2}{\sqrt{2}}\sqrt{x} = \sqrt{2}\sqrt{x} & \text{for } 0 \leq x \leq \frac{1}{2} \\ 1 - x - (-\sqrt{\frac{x}{2}}) = 1 - x + \frac{1}{\sqrt{2}}\sqrt{x} & \text{for } \frac{1}{2} \leq x \leq 2 \end{cases}$$

$$\text{So } A = \int_0^2 h(x) dx = \int_0^{\frac{1}{2}} h(x) dx + \int_{\frac{1}{2}}^2 h(x) dx$$

$$= \int_0^{\frac{1}{2}} \sqrt{2}\sqrt{x} dx + \int_{\frac{1}{2}}^2 \left(1 - x + \frac{1}{\sqrt{2}}\sqrt{x}\right) dx = \sqrt{2} \int_0^{\frac{1}{2}} x^{\frac{1}{2}} dx + \int_{\frac{1}{2}}^2 (1 - x) dx$$

$$+ \frac{1}{\sqrt{2}} \int_{\frac{1}{2}}^2 \sqrt{x} dx = \sqrt{2} \frac{2}{3} x^{\frac{3}{2}} \Big|_0^{\frac{1}{2}} + \left\{ -\frac{(1-x)^2}{2} \right\} \Big|_{\frac{1}{2}}^2 + \frac{1}{\sqrt{2}} \frac{2}{3} x^{\frac{3}{2}} \Big|_{\frac{1}{2}}^2 =$$

$$\sqrt{2} \left(\frac{2}{3} \frac{1}{2\sqrt{2}} - 0 \right) + \left\{ -\frac{1}{2}(1-2)^2 + \frac{1}{2}\left(1-\frac{1}{2}\right)^2 \right\} + \frac{\sqrt{2}}{3} \left\{ 2\sqrt{2} - \frac{1}{2\sqrt{2}} \right\} =$$

$$\frac{1}{3} - \frac{1}{2} + \frac{1}{8} + \frac{4}{3} - \frac{1}{6} = \frac{9}{8} \text{ (perforce)}$$