1 (a) $$\frac{dy}{dx} = 1\cdot\sin\left(\frac{1}{x}\right) + x\frac{d}{dx}\sin\left(\frac{1}{x}\right) = \sin\left(\frac{1}{x}\right) + x\cos\left(\frac{1}{x}\right)\frac{d}{dx}\left(x^{-1}\right)$$
$$= \sin\left(\frac{1}{x}\right) + x\cos\left(\frac{1}{x}\right)\left(-x^{-2}\right) = \sin\left(\frac{1}{x}\right) - \frac{1}{x}\cos\left(\frac{1}{x}\right)$$
$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=3/\pi} = \sin\left(\frac{\pi}{3}\right) - \frac{\pi}{3}\cos\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{3}\cdot\frac{1}{2} = \frac{3\sqrt{3}-\pi}{6}$$

(b) $$y^2 = \frac{x^3-1}{x^3+1} \Rightarrow 2y\frac{dy}{dx} = \frac{(x^3+1)(3x^2-0)-(x^3-1)(3x^2+0)}{(x^3+1)^2} = \frac{6x^2}{(x^3+1)^2}$$
$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=2} = \left.\frac{3x^2}{y(x^3+1)^2}\right|_{x=2} = \frac{12}{\sqrt{\frac{7}{9}}\,81} = \frac{4}{9\sqrt{7}}$$

(c) $$\frac{dy}{dx} = -4e^{-4x}\cos(7x) + e^{-4x}\{-7\sin(7x)\} \Rightarrow$$
$$\left.\frac{dy}{dx}\right|_{x=\pi/4} = -4e^{-\pi}\cos(7\pi/4) - 7e^{-\pi}\sin(7\pi/4) = -4e^{-\pi}\left(\frac{1}{\sqrt{2}}\right) - 7e^{-\pi}\left(\frac{-1}{\sqrt{2}}\right) = \frac{3e^{-\pi}}{\sqrt{2}}$$

(d) $$y = e^{\cos(x)\ln(\ln(x))}$$
$$\Rightarrow \frac{dy}{dx} = e^{\cos(x)\ln(\ln(x))}\frac{d}{dx}\left\{\cos(x)\ln(\ln(x))\right\}$$

WITH $u = \ln(x)$, $\frac{d}{dx}[\ln(u)] = \frac{1}{u}\frac{du}{dx}$

$$= \ln(x)^{\cos(x)}\left\{-\sin(x)\ln(\ln(x)) + \cos(x)\frac{1}{\ln(x)}\frac{d}{dx}(\ln(x))\right\}$$
$$= \ln(x)^{\cos(x)}\left\{-\sin(x)\ln(\ln(x)) + \cos(x)\frac{1}{\ln(x)}\cdot\frac{1}{x}\right\},\ x>0$$
$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=e} = \ln(e)^{\cos(e)}\left\{-\sin(e)\ln(\ln(e)) + \cos(e)\frac{1}{\ln(e)}\cdot\frac{1}{e}\right\}$$
$$= 1^{\cos(e)}\left\{-\sin(e)\ln(1) + \cos(e)\cdot\frac{1}{1}\cdot\frac{1}{e}\right\}$$
$$= 1\cdot\left\{-\sin(e)\cdot 0 + \frac{\cos(e)}{e}\right\} = \frac{\cos(e)}{e}$$

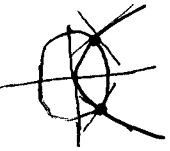
2 (a) $$x^2 + 2xy - y^2 + x = 2 \Rightarrow 2x + 2\frac{d}{dx}[xy] - \frac{d}{dx}[y^2] + 1 = \frac{d}{dx}[2]$$
$$\Rightarrow 2x + 2\left\{1\cdot y + x\frac{dy}{dx}\right\} - 2y\frac{dy}{dx} + 1 = 0$$
$$\Rightarrow 2x + 2y + 1 = 2(y-x)\frac{dy}{dx}.$$
So when $x=1$ and $y=2$ we have $2+4+1 = 2(2-1)y'$ or $y' = \frac{7}{2}$. Therefore, the tangent has slope $\frac{7}{2}$ and hence equation $y-2 = \frac{7}{2}(x-1)$ or $7x-2y = 3$

(b) $$x^{1/2} + y^{1/2} = 1 \Rightarrow \frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{y^{1/2}}{x^{1/2}}$$
$$= -\left(\frac{y}{x}\right)^{1/2} \Rightarrow y'' = \frac{d}{dx}(y') = -\frac{1}{2}\left(\frac{y}{x}\right)^{-1/2}\frac{d}{dx}\left(\frac{y}{x}\right)$$
$$= -\frac{1}{2}\left(\frac{x}{y}\right)^{1/2}\left\{\frac{xy' - y\cdot 1}{x^2}\right\} = -\frac{1}{2x^2}\frac{x^{1/2}}{y^{1/2}}\left\{x\left(-\frac{y^{1/2}}{x^{1/2}}\right) - y\right\}$$

$$= \frac{1}{2x^{3/2}y^{1/2}}\left\{ x^{1/2}y^{1/2} + y \right\} = \frac{1}{2x^{3/2}}\left(x^{1/2}+y^{1/2}\right) = \frac{1}{2x^{3/2}} \Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=c^2} = \frac{1}{2c^3}$$

3 (a) $x = y^2$ meets $2x^2 + y^2 = 3$ where $2x^2 + x - 3 = 0 \Rightarrow$ $(2x+3)(x-1) = 0 \Rightarrow x = 1$ because $x = -3/2$ is impossible because $x = y^2 \geq 0$. So the curves meet at $(1,-1)$ and $(1,1)$.

From $x = y^2$ we have $1 = 2yy' \Rightarrow y'_P = \frac{1}{2y}$, where P denotes parabola.



From $2x^2+y^2=3$ we have $4x + 2yy' = 0 \Rightarrow y'_E = -\frac{4x}{2y} = \frac{-2x}{y}$, where E denotes ellipse. So where the curves cross we have

$$y'_P\, y'_E = \frac{1}{2y}\cdot\frac{-2x}{y} = \frac{-x}{y^2} = -1$$ regardless of which point of intersection

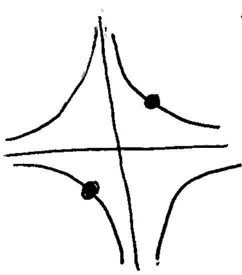
The curves are therefore orthogonal.

(b) $x^2y^2 + xy = 2 \Rightarrow 2xy^2 + x^2\frac{d}{dx}(y^2) + 1\cdot y + x\frac{dy}{dx} = 0$

$\Rightarrow 2xy^2 + x^2\cdot 2yy' + y + xy' = 0$

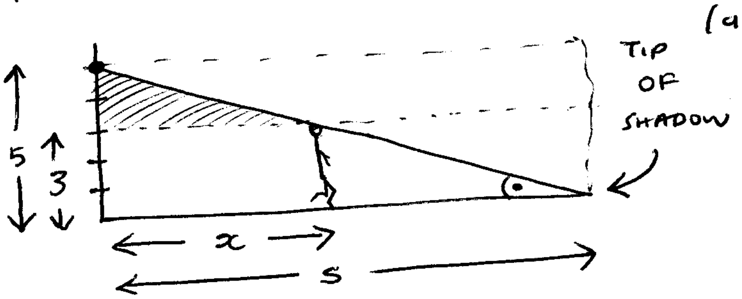
$\Rightarrow xy'(1+2xy) + y(1+2xy) = 0 \Rightarrow (xy'+y)(1+2xy) = 0$

Now, we can't satisfy this equation with $1+2xy = 0$ because $xy = -\frac{1}{2} \Rightarrow x^2y^2 + xy = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$, which does not equal 2.



So $xy' + y = 0 \Rightarrow y' = -\frac{y}{x}$. Therefore, if $y' = -1$ then $y = x$ and $x^4 + x^2 = 2 \Rightarrow (x^2)^2 + (x^2) - 2 = 0 \Rightarrow (x^2+2)(x^2-1) = 0$ $\Rightarrow x^2 = 1 \Rightarrow x = \pm 1$. Hence the slope is -1 at $(-1,-1)$ and $(1,1)$.

4.



(a) The biggest triangle is similar to the one whose interior is hatched. Hence

$$\frac{5-3}{x} = \frac{5}{s} \quad (= \tan(\cdot))$$

$$\Rightarrow s = \frac{5x}{2}$$

$$\Rightarrow \frac{ds}{dt} = \frac{5}{2}\frac{dx}{dt} = \frac{5}{2}\cdot 2 = 5$$

(b) Clearly, no.

5. For $f(x) = x^3 - 6x^2 + 9x + 2$ we have $f'(x) = 3x^2 - 12x + 9$ $= 3(x^2 - 4x + 3) = 3(x-1)(x-3)$. So the only critical points are $x=1$ and $x=3$. Hence the only extremizer candidates are $x=-1$, $x=4$ (the endpoints) and $x=1$, $x=3$. From the table we see that there is a unique global minimizer $(x=-1)$ but there are two global maximizers $(x=1,4)$; the global max and min are $6, -14$, respectively.

x	$f(x)$
-1	-14
1	6
3	2
4	6