MAC 2311, Sections 07 & 09, Fall 2004 (Dr M-G)

Mock Second Test

Tuesday, October 5, 2004

You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test. You may assume

$\frac{d}{dx}\left\{\sinh^{-1}(x)\right\} = \frac{1}{\sqrt{x^2+1}}, \quad \frac{d}{dx}\left\{\cosh^{-1}(x)\right\} = \frac{1}{\sqrt{x^2-1}}, \quad \frac{d}{dx}\left\{\tanh^{-1}(x)\right\} = \frac{1}{1-x^2}$

(for $x > 1$ in the second case and $-1 < x < 1$ in the third).

DO NOT WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT NOT STAPLED TO YOUR SOLUTIONS)

1. In each of the following cases, find the *exact* value of $m = \left.\frac{dy}{dx}\right|_{\substack{x=a\\y=b}}$ for the given a, b:

(a) $y\sin(x^2) = x\sin(y^2)$, $a = \frac{1}{2}\sqrt{\pi}$, $b = \frac{1}{2}\sqrt{\pi}$ [6]

(b) $y = \sqrt[4]{\frac{x^2-1}{x^2+1}}$, $a = \sqrt{\frac{17}{15}}$, $b = \frac{1}{2}$ [6]

2. What is the tangent line to the curve $y = \ln(\ln(x))$ at the point $(e, 0)$? [5]

3. The position at time t of a particle moving along the x-axis is given by $x = \dfrac{t}{1+t^2}$.

(a) Find the particle's velocity at time t. [4]

(b) Find the particle's acceleration at time t. [6]

(c) Describe its motion for $t \geq 0$. [2]

4. Find both $m = \left.\frac{dy}{dx}\right|_{(x,y)=(8,27)}$ and $\alpha = \left.\frac{d^2y}{dx^2}\right|_{(x,y)=(8,27)}$ for $\sqrt[3]{x} + \sqrt[3]{y} = 5$. [8]

5. In each of the following cases, reduce $\frac{dy}{dx}$ to its simplest possible form:

(a) $y = \sqrt{9-x^2}\,\sinh^{-1}(2x)$. [3]

(b) $y = \ln(x^6\sin^4(x))$. [3]

(c) $y = \{\ln(1+e^{x^2})\}^4$. [3]

(d) $y = x\,\tanh^{-1}(x) + \ln(\sqrt{3-x^4})$, $-1 < x < 1$. [3]

(e) $y = x^4\,\sinh^{-1}(4x)$. [3]

6. Find equations for both tangent lines to the ellipse $x^2 + 4y^2 = 36$ that pass through the point $(12, 3)$. [8]

[Perfect score: $12 + 5 + 12 + 8 + 15 + 8 = 60$]