1 (a) Because polynomials are continuous everywhere, and $2^3 + 2\cdot 2 = 12 = 2^2 - 9\cdot 2^3 + 5\cdot 2^4 \Rightarrow g(2-) = g(2+)$.

(b) $\int_1^3 g(t)dt = \int_1^2 g(t)dt + \int_2^3 g(t)dt = \int_1^2 (t^3 + 2t)dt + \int_2^3 (t^2 - 9t^3 + 5t^4)dt = \left\{\frac{1}{4}t^4 + t^2\right\}\Big|_1^2 + \left\{\frac{1}{3}t^3 - \frac{9}{4}t^4 + t^5\right\}\Big|_2^3$
$= \frac{1}{4}2^4 + 2^2 - \frac{1}{4}1^4 - 1^2 + \frac{1}{3}3^3 - \frac{9}{4}3^4 + 3^5 - \frac{1}{3}2^3 + \frac{9}{4}2^4 - 2^5$
$= 4 + 4 - \frac{1}{4} - 1 + 9 - \frac{729}{4} + 243 - \frac{8}{3} + 36 - 32 = \frac{467}{6}$

2 By the fundamental theorem, $F'(t) = f(t)$. So

$$F'(t) = \begin{cases} 3 + 2t & \text{if } t \in [0,2) \\ 13 - 3t & \text{if } t \in [2,4) \\ t - 3 & \text{if } t \in [4,5) \\ 2 & \text{if } t \in [5,7] \end{cases} \Rightarrow F(t) = \begin{cases} 3t + t^2 + c_1 & \text{if } t \in [0,2) \\ 13t - \frac{3}{2}t^2 + c_2 & \text{if } t \in [2,4) \\ \frac{1}{2}t^2 - 3t + c_3 & \text{if } t \in [4,5) \\ 2t + c_4 & \text{if } t \in [5,7] \end{cases}$$

where c_1, c_2, c_3 and c_4 are constants. But clearly $F(0) = 0 \Rightarrow 3\cdot 0 + 0^2 + c_1 = 0 \Rightarrow c_1 = 0$; and F must be continuous, implying $F(2-) = F(2+)$ or $3\cdot 2 + 2^2 + c_1 = 13\cdot 2 - \frac{3}{2}2^2 + c_2$, $F(4-) = F(4+)$ or $13\cdot 4 - \frac{3}{2}\cdot 4^2 + c_2 = \frac{1}{2}4^2 - 3\cdot 4 + c_3$ and $F(5-) = F(5+)$ or $\frac{1}{2}5^2 - 3\cdot 5 + c_3 = 2\cdot 5 + c_4$. Hence $c_2 = 6 + 4 + 0 - 26 + 6 = -10$; $c_3 = 52 - 24 + c_2 - 8 + 12 = 22$ and $c_4 = 25/2 - 15 + c_3 - 10 = \frac{19}{2}$. Therefore

$$F(t) = \begin{cases} 3t + t^2 & \text{if } 0 \le t < 2 \\ 13t - \frac{3}{2}t^2 - 10 & \text{if } 2 \le t < 4 \\ \frac{1}{2}t^2 - 3t + 22 & \text{if } 4 \le t < 5 \\ 2t + 19/2 & \text{if } 5 \le t \le 7 \end{cases}$$

3. $u = \sqrt{2x+3} \Rightarrow u^2 = 2x + 3 \Rightarrow x = \frac{1}{2}u^2 - 3/2 \Rightarrow \frac{dx}{du} = \frac{1}{2}\cdot 2u - 0 = u$. So $I = \int_{x=1/2}^{x=3} \frac{7x+6}{(\sqrt{2x+3})^7}dx$

$$= \int_{u=\sqrt{2\cdot 1/2+3}}^{u=\sqrt{2\cdot 3+3}} \frac{7\left(\frac{1}{2}u^2 - 3/2\right) + 6}{u^7}\frac{dx}{du}du$$

$$= \int_2^3 \frac{\frac{7}{2}u^2 - \frac{9}{2}}{u^7}u\,du = \int_2^3 \left(\frac{7}{2}u^{-4} - \frac{9}{2}u^{-6}\right)du$$

$$= \left\{\frac{7}{2}\left\{\frac{-1}{3}u^{-3}\right\} - \frac{9}{2}\left\{\frac{-1}{5}u^{-5}\right\}\right\}\Bigg|_2^3 = -\frac{7}{6}3^{-3} + \frac{9}{10}3^{-5}$$
$$+ \frac{7}{6}2^{-3} - \frac{9}{10}2^{-5} = \frac{-7}{162} + \frac{9}{2430} + \frac{7}{48} - \frac{9}{320} = \frac{2027}{25920}$$

4 (a) Let $g(x) = \left(x - \frac{2}{\sqrt{x}}\right)^4 = x^4 + 4x^3\left(\frac{-2}{\sqrt{x}}\right) + 6x^2\left(\frac{-2}{\sqrt{x}}\right)^2 + 4x\left(\frac{-2}{\sqrt{x}}\right)^3 + \left(\frac{-2}{\sqrt{x}}\right)^4$

$= x^4 - 8x^{5/2} + 24x - 32x^{-1/2} + 16x^{-2}$. Then

$$\int_1^2 g(x)\,dx = \int_1^2 x^4\,dx - 8\int_1^2 x^{5/2}\,dx + 24\int_1^2 x\,dx - 32\int_1^2 x^{-1/2}\,dx + 16\int_1^2 x^{-2}\,dx$$

$$= \frac{x^5}{5}\Big|_1^2 - 8\cdot\frac{2}{7}x^{7/2}\Big|_1^2 + 24\cdot\frac{x^2}{2}\Big|_1^2 - 32\cdot 2x^{1/2}\Big|_1^2 + 16\{-x^{-1}\}\Big|_1^2$$

$$= \frac{1}{5}(2^5 - 1^5) - \frac{16}{7}(2^{7/2} - 1^{7/2}) + 12(2^2 - 1^2) - 64(2^{1/2} - 1) + 16\left(-\frac{1}{2} + 1\right)$$

$$= \frac{31}{5} - \frac{128}{7}\sqrt{2} + \frac{16}{7} + 36 - 64\sqrt{2} + 64 + 8 = \frac{4077}{35} - \frac{576\sqrt{2}}{7}$$

(b). Let $g(x) = (3-4x)(4-3x)(5-2x) = (3-4x)\{20 - 23x + 6x^2\}$

$= 60 - 149x + 110x^2 - 24x^3$.

Then, because x and x^3 are both odd, whereas 1 and x^2 are both even, $\int_{-1}^1 g(x)\,dx = 60\int_{-1}^1 1\,dx - 149\int_{-1}^1 x\,dx + 110\int_{-1}^1 x^2\,dx - 24\int_{-1}^1 x^3\,dx$

$$= 120\int_0^1 1\,dx - 149\cdot 0 + 220\int_0^1 x^2\,dx - 24\cdot 0$$

$$= 120(1-0) - 0 + 220\left(\frac{1^3 - 0^3}{3}\right) = \frac{580}{3}$$

(c) $|e^{2x} - 2| = \begin{cases} e^{2x} - 2 & \text{if } e^{2x} \geq 2 \\ 2 - e^{2x} & \text{if } e^{2x} < 2 \end{cases}$

But $e^{2x} \geq 2 \Leftrightarrow \ln(e^{2x}) \geq \ln(2) \Leftrightarrow 2x \geq \ln(2) \Leftrightarrow x \geq \frac{\ln(2)}{2}$.

Hence $|e^{2x} - 2| = \begin{cases} 2 - e^{2x} & \text{if } x < \frac{1}{2}\ln(2) \\ e^{2x} - 2 & \text{if } x \geq \frac{1}{2}\ln(2) \end{cases}$

$$\Rightarrow \int_0^1 |e^{2x} - 2|\,dx = \int_0^{\frac{1}{2}\ln(2)} (2 - e^{2x})\,dx + \int_{\frac{1}{2}\ln(2)}^1 (e^{2x} - 2)\,dx$$

$$= \int_0^{\frac{1}{2}\ln(2)} \frac{d}{dx}\left\{2x - \frac{1}{2}e^{2x}\right\}dx + \int_{\frac{1}{2}\ln(2)}^1 \frac{d}{dx}\left\{\frac{1}{2}e^{2x} - 2x\right\}dx$$

$$= \left(2x - \frac{1}{2}e^{2x}\right)\Big|_0^{\frac{1}{2}\ln(2)} + \left(\frac{1}{2}e^{2x} - 2x\right)\Big|_{\frac{1}{2}\ln(2)}^1$$

$$= \ln(2) - 1 - \left\{0 - \frac{1}{2}\right\} + \frac{1}{2}e^2 - 2 - \{1 - \ln(2)\}$$

$$= \frac{1}{2}e^2 + 2\ln(2) - \frac{7}{2}$$

5

$$f''(t) = \frac{15}{4}\left\{\frac{5t(5t^2+6t+9) - 3(5t^2+6t+9)}{t^{7/2}}\right\}$$

$$= \frac{15}{4t^{7/2}}\left\{25t^3 + 15t^2 + 27t - 27\right\}$$

$$= \frac{15}{4}\left\{25t^{-1/2} + 15t^{-3/2} + 27t^{-5/2} - 27t^{-7/2}\right\}$$

$$\Rightarrow f'(t) = \int \frac{15}{4}\left\{25t^{-1/2} + 15t^{-3/2} + 27t^{-5/2} - 27t^{-7/2}\right\}dt$$

$$= \frac{15}{4}\left\{25\frac{t^{1/2}}{1/2} + 15\frac{t^{-1/2}}{(-1/2)} + 27\frac{t^{-3/2}}{(-3/2)} - 27\frac{t^{-5/2}}{(-5/2)}\right\}$$

$$+ c$$

But $f'(1) = 48 \Rightarrow \frac{15}{4}\left\{50 - 30 - 18 + \frac{54}{5}\right\} + c = 48$

$$\Rightarrow \frac{15}{4}\frac{64}{5} + c = 48 \Rightarrow 48 + c = 48 \Rightarrow c = 0$$

So $f'(t) = \frac{15}{4}\left\{50t^{1/2} - 30t^{-1/2} - 18t^{-3/2} + \frac{54t^{-5/2}}{5}\right\}$

$$\Rightarrow f(t) = \frac{15}{4}\left\{\frac{100}{3}t^{3/2} - 60t^{1/2} + 36t^{-1/2} - \frac{36}{5}t^{-3/2}\right\} + b$$

But $f(1) = 8 \Rightarrow \frac{15}{4}\left\{\frac{100}{3} - 60 + 36 - \frac{36}{5}\right\} + b = 8 \Rightarrow$
$\frac{15}{4} \cdot \frac{32}{15} + b = 8 \Rightarrow 8 + b = 8 \Rightarrow b = 0.$ So

$$f(t) = \frac{15}{4}\left\{\frac{100}{3}t\sqrt{t} - 60\sqrt{t} + \frac{36}{\sqrt{t}} - \frac{36}{5}\cdot\frac{1}{t\sqrt{t}}\right\}$$

$$= 125t\sqrt{t} - 225\sqrt{t} + \frac{135}{\sqrt{t}} - \frac{27}{t\sqrt{t}}$$

$$= (5\sqrt{t})^3 + 3(5\sqrt{t})^2\left(-\frac{3}{\sqrt{t}}\right) + 3\cdot 5\sqrt{t}\left(-\frac{3}{\sqrt{t}}\right)^2 + \left(-\frac{3}{\sqrt{t}}\right)^3$$

$$= \left(5\sqrt{t} - \frac{3}{\sqrt{t}}\right)^3$$

6.

$$\int_1^b x^{-4}\,dx = \frac{7}{24} \Rightarrow \int_1^b \frac{d}{dx}\left(-\frac{1}{3}x^{-3}\right)dx = \frac{7}{24} \Rightarrow$$

$$-\frac{1}{3}x^{-3}\Big|_1^b = \frac{7}{24} \Rightarrow -\frac{1}{3}\left(b^{-3} - 1^{-3}\right) = \frac{7}{24} \Rightarrow$$

$$\frac{1}{b^3} - 1 = \frac{-7}{8} \Rightarrow \frac{1}{b^3} = \frac{1}{8} \Rightarrow b^3 = 8 \Rightarrow b = 2.$$