MAC 2311, Sections 07 & 09, Fall 2004 (Dr M-G)

Mock Fourth Test

Thursday, November 18, 2004

You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test.

DO NOT WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT NOT STAPLED TO YOUR SOLUTIONS)

1. The function f is defined on $[-\frac{13}{3}, \frac{25}{3}]$ by

$$f(x) \;=\; (x-1)(x^3 - 11x^2 + x + 225).$$

(a) Find all local extremizers of f on $[-\frac{13}{3}, \frac{25}{3}]$. [12]

(b) Find both $\max(f, -\frac{13}{3}, \frac{25}{3})$ and $\min(f, -\frac{13}{3}, \frac{25}{3})$. [4]

2. **(a)** A non-Norman window consists of a rectangle surmounted by a right-angled isosceles triangle, with the right angle at the apex of the window. If the window's perimeter is fixed, find the ratio of height to width that admits the greatest possible amount of light. [8]

(b) Find the area of the largest rectangle inscribable in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [8]

3. Use L'Hôpital's rule to calculate $\lim\limits_{x \to 0} f(x)$ where:

(a) $f(x) \;=\; \dfrac{x^3}{x - \sin(x)}$ [6]

(b) $f(x) \;=\; \dfrac{x - \ln(1+x)}{x \ln(1+x)}$ [6]

4. Find the *exact* area of the region bounded by the curves $y = 16\sqrt[3]{x}$ and $x^3 = 16y$

(a) by integrating with respect to x and [6]

(b) by integrating with respect to y. [6]

5. The region bounded by $y = 16\sqrt[3]{x}$ and $x^3 = 16y$ is rotated (through angle 2π) about the axis of symmetry $x = -1$. Find the *exact* volume of the solid thus generated

(a) by integrating with respect to x and [12]

(b) by integrating with respect to y. [12]

[Perfect score: $2 \times 16 + 2 \times 12 + 24 = 80$]