MAC 2311, Section 02, Summer 2004 (Dr M-G)

First Test

Wednesday, June 2, 2004

You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have one hour. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test. You may assume that

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

for any value of n (regardless of whether it is positive or negative, and regardless of whether it is a fraction or an integer).

1. In each of the following cases, either find the limit or state that it doesn't exist:

(a) $\lim\limits_{x\to 2} \frac{x^2+x-6}{x^2-4}$ [3] **(b)** $\lim\limits_{x\to -2^-} \frac{x^2+x-6}{x^2-4}$ [1] **(c)** $\lim\limits_{x\to -2^+} \frac{x^2+x-6}{x^2-4}$ [1]

(d) $\lim\limits_{x\to -2} \frac{x^2+x-6}{x^2-4}$ [1] **(e)** $\lim\limits_{x\to 3} \frac{x^2+x-6}{x^2-4}$ [1] **(f)** $\lim\limits_{x\to -\infty} \frac{x^2+x-6}{x^2-4}$ [1]

(g) $\lim\limits_{x\to \infty} \frac{x^2+x-6}{x^2-4}$ [1] **(h)** $\lim\limits_{x\to \infty} \sqrt{x^2+1}-\sqrt{x^2-3x+1}$ [3]

2. In each of the following cases, find $f'(x)$ *from first principles* (i.e., by using the definition of derivative in terms of a limit):

(a) $f(x) = \frac{1}{\sqrt{2x-3}}$ [6] **(b)** $f(x) = \frac{x+2}{x+1}$ [6]

In each case, state the domain of both f and f'.

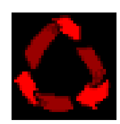
3. In each of the following cases, use the product or quotient rule to find $h'(1)$:

(a) $h(x) = (x^4-3x+4)(e^x+x^3+2)$ [6] **(b)** $h(x) = \frac{x^4-3x+4}{e^x+x^3+2}$ [6]

In each case, simplify your answer as much as possible.

4. A function is said to be *smooth* on $(-\infty, \infty)$ if both it and its derivative are continuous everywhere. For what values of a and b is f defined by

$$f(x) = \begin{cases} a+bx & \text{if} \quad x \le 2 \\ \frac{x+2}{x+1} & \text{if} \quad x > 2 \end{cases}$$



a smooth function? [7]

Hint: Recycle the result you obtained for Question **2(b)**.

5. **(a)** Find an equation of the tangent line to $y = \frac{9x^2}{1+2x^2}$ at the point $(2, 4)$. [4]

(b) This line meets the curve in another point. Find it. [3]

[Perfect score: $3 \times 12 + 2 \times 7 = 50$]