

### Fourth Test

Wednesday, August 4, 2004

**You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have one hour. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test. You may assume that**

$$\int 2(1+y) \ln\left(\frac{1}{y}\right) dy = y(y+2) \ln\left(\frac{1}{y}\right) + 2y + \frac{1}{2}y^2 + C$$

**(where  $C$  is an arbitrary constant).**

1. For  $y = \frac{(1+5x^4)^{\frac{9}{4}}}{(5+x^2)^3}$  find the *exact* value of  $\left.\frac{dy}{dx}\right|_{x=2}$ . [12]

**Note:** The correct answer exceeds 80 but is less than 90.

2. Use L'Hôpital's rule to calculate  $\lim_{x \rightarrow 0} f(x)$  where  $f(x) = \frac{1}{x} - \frac{1}{2 \ln\left(1 + \frac{1}{2}x\right)}$ . [12]

**Note:** The correct answer is negative but exceeds -1.

3. Use the substitution  $u = \sqrt{1+4x}$  to calculate  $I = \int_0^2 \frac{3x+1}{\sqrt{1+4x}} dx$ . [12]

**Note:** The correct answer exceeds 3 but is less than 4.

4. The region bounded by the lines  $x = 0$ ,  $x = 1$ ,  $y = 0$  and the curve  $y = e^{-x}$  or  $x = \ln(1/y)$  is rotated (through angle  $2\pi$ ) about the axis of symmetry  $y = -1$ . Find the *exact* volume of the solid thus generated

(a) by integrating with respect to  $x$  and [6]

(b) by integrating with respect to  $y$ . [8]

**Note:** The correct answer exceeds 5 but is less than 6.