1. $\ln(y) = \frac{9}{4}\ln\left(1+5x^4\right) - 3\ln\left(5+x^2\right) \Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{9}{4}\cdot\frac{1}{1+5x^4}\cdot 20x^3$

$- 3\cdot\frac{1}{5+x^2}\cdot 2x \Rightarrow \left.\frac{dy}{dx}\right|_{x=2} = y\Big|_{x=2}\left(\frac{9}{4}\cdot\frac{1}{1+80}\cdot 20\cdot 8 - 3\cdot\frac{1}{9}\cdot 4\right) =$

$\frac{(1+80)^{9/4}}{9^3}\left\{\frac{9}{4}\cdot\frac{1}{81}\cdot 20\cdot 8 - \frac{4}{3}\right\} = \frac{3^9}{9^3}\left\{\frac{40}{9} - \frac{12}{9}\right\} = \frac{3^9}{3^6}\cdot\frac{28}{3^2} = 3\cdot 28 = \underline{\underline{84}}$

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2. $f(x) = \frac{g(x)}{h(x)}$ where $g(x) = 2\ln\left(1+\tfrac{1}{2}x\right) - x$, $h(x) = 2x\ln\left(1+\tfrac{1}{2}x\right)$

$\Rightarrow g'(x) = \frac{2}{1+\frac{1}{2}x}\cdot\frac{1}{2} - 1$, $h'(x) = 2\ln\left(1+\tfrac{1}{2}x\right) + \frac{2x}{1+\frac{1}{2}x}\cdot\frac{1}{2}$

$\Rightarrow g''(x) = -\frac{1}{\left(1+\frac{1}{2}x\right)^2}\cdot\frac{1}{2}$, $h''(x) = \frac{2}{1+\frac{1}{2}x}\cdot\frac{1}{2} + \frac{\left(1+\frac{1}{2}x\right)\cdot 1 - x\cdot\frac{1}{2}}{\left(1+\frac{1}{2}x\right)^2}$

So $g(0) = 0$, $g'(0) = 1-1 = 0$, $g''(0) = -\frac{1}{2}$, $h(0) = 0$, $h'(0) = 0$ and $h''(0) = 1+1 = 2$. Therefore $\frac{g(0)}{h(0)}$ and $\frac{g'(0)}{h'(0)}$ are both of form $\frac{0}{0}$, yielding

$\lim_{x\to 0} f(x) = \lim_{x\to 0}\frac{g'(x)}{h'(x)} = \lim_{x\to 0}\frac{g''(x)}{h''(x)} = \frac{g''(0)}{h''(0)} = \frac{-1/2}{2} = \underline{\underline{\frac{-1}{4}}}$

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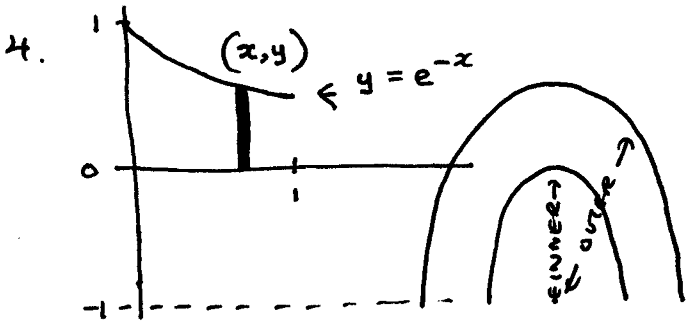
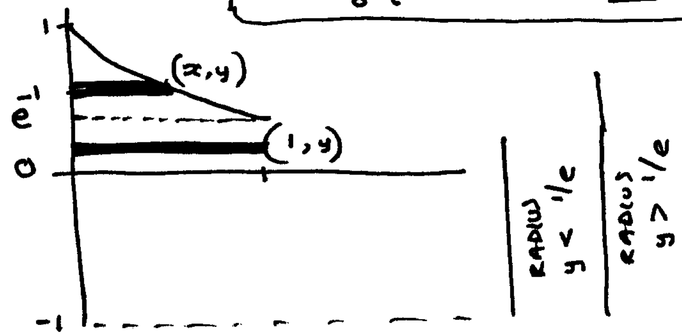
3. $u = \sqrt{1+4x} \Rightarrow u^2 = 1+4x \Rightarrow x = \frac{1}{4}(u^2-1) \Rightarrow \frac{dx}{du} = \frac{1}{4}\{2u - 0\} = \frac{1}{2}u$. So

$I = \int_{u=\sqrt{1+0}}^{u=\sqrt{1+8}} \frac{3x+1}{\sqrt{1+4x}}\frac{dx}{du}\,du = \int_1^3 \frac{\frac{3}{4}(u^2-1)+1}{u}\,\frac{1}{2}u\,du = \frac{1}{8}\int\left(3u^2+1\right)du = \left.\frac{u^3+u}{8}\right|_1^3$

$= \frac{1}{8}\{30-2\} = \underline{\underline{\frac{7}{2}}}$

4.





(a) $\delta V \approx \pi\left\{(1+y)^2 - 1^2\right\}\delta x$

$\Rightarrow V = \int_{x=0}^{x=1}\pi\left\{(1+y)^2 - 1\right\}dx$

$= \pi\int_0^1\left(y^2+2y\right)dx = \pi\int_0^1\left(e^{-2x}+2e^{-x}\right)dx$

$= \pi\left\{-\frac{1}{2}e^{-2x} - 2e^{-x}\right\}\Big|_0^1$

$= \pi\left\{-\frac{1}{2}e^{-2} - 2e^{-1} + \frac{1}{2} + 2\right\}$

$= \underline{\underline{\frac{1}{2}\left(5 - \frac{4}{e} - \frac{1}{e^2}\right)\pi}}$

(b) $\delta V \approx 2\pi$ radius "height" δy

$= \begin{cases} 2\pi(1+y)x\,\delta y & \text{if } \frac{1}{e}\le y\le 1 \\ 2\pi(1+y)\cdot 1\,\delta y & \text{if } 0\le y\le 1/e \end{cases}$

So $V = \int_{y=0}^{y=1/e} 2\pi(1+y)\,dy + \int_{y=1/e}^{y=1} 2\pi(1+y)x\,dy$

$= \int_0^{1/e} 2\pi(1+y)\,dy + \int_{1/e}^1 2\pi(1+y)\ln\left(\frac{1}{y}\right)dy$

$= \pi(1+y)^2\Big|_0^{1/e} + \pi\left\{y(y+2)\ln\left(\frac{1}{y}\right) + 2y + \frac{1}{2}y^2\right\}\Big|_{1/e}^1$

$= \pi\left\{\left(1+\frac{1}{e}\right)^2 - 1\right\} + \pi\left\{3\ln(1) + \frac{5}{2} - \frac{1}{e}\left(\frac{1}{e}+2\right)\ln(e) - \frac{2}{e} - \frac{1}{2e^2}\right\} = \frac{1}{2}\left(5 - \frac{4}{e} - \frac{1}{e^2}\right)\pi$ after simplification