

Fourth Test

Thursday, November 18, 2004

You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test.

DO NOT WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT NOT STAPLED TO YOUR SOLUTIONS)

1. The function f is defined on $[-3, 3]$ by

$$f(x) = (x - 1)(2x^2 - x - 13).$$

- (a) Find all local extremizers of f on $[-3, 3]$. [12]
 (b) Find both $\max(f, -3, 3)$ and $\min(f, -3, 3)$. [4]

2. Find the *exact* area of the region bounded by the curves $y^2 = x$ and $x^2 = y$. [10]

3. The region bounded by $y^2 = x$ and $x^2 = y$ is rotated (through angle 2π) about the axis of symmetry $y = -1$. Find the *exact* volume of the solid thus generated

- (a) by integrating with respect to x and [12]
 (b) by integrating with respect to y . [12]

4. Use L'Hôpital's rule to calculate $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{\sin(x) - \ln(1 + x)}$. [14]



5. A square cookie cutter and a circular cookie cutter are to be made from a thin strip of metal of length L by cutting it into two pieces and bending each piece appropriately. If the sum of the areas of the two resulting cookies must be as small as possible (perhaps to ensure that whoever eats them ingests as few calories as possible) then
- (a) How long is a side of the square cookie? [6]
 (b) What is the radius of the circular cookie? [6]
 (c) What is the ratio of the larger cookie's area to the area of the smaller one? [4]