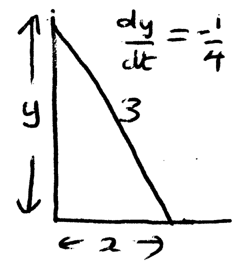
1. $y^2 = (x^2+3)e^{2x} \Rightarrow 2y\frac{dy}{dx} = (2x+0)e^{2x} + (x^2+3)\cdot 2e^{2x} \Rightarrow 2\cdot 2e\,\frac{dy}{dx}\Big|_{x=1} = 2e^2+8e^2$

$\Rightarrow \frac{dy}{dx}\Big|_{x=1} = \frac{5e}{2}$. So tangent line is $y - 2e = \frac{5e}{2}(x-1)$ or $y = \frac{1}{2}e(5x-1)$.


2. (a) $\int_{1/2}^{1}\left(2x+\frac{1}{x}\right)dx = \left\{x^2+\ln(x)\right\}\Big|_{1/2}^{1} = 1+\ln(1)-\frac{1}{4}-\ln\left(\frac{1}{2}\right) = \ln(2)+\frac{3}{4}$

(b) $\int_{1/2}^{1} 2\pi x\left(2x+\frac{1}{x}\right)dx = 2\pi\int_{1/2}^{1}(2x^2+1)dx = 2\pi\left(\frac{2}{3}x^3+x\right)\Big|_{1/2}^{1}$
$= 2\pi\left\{\frac{2}{3}+1-\frac{1}{12}-\frac{1}{2}\right\} = \frac{13\pi}{6}$

(c) $\int_{1/2}^{1}\pi\left(2x+\frac{1}{x}\right)^2dx = \pi\int_{1/2}^{1}\left(4x^2+4+\frac{1}{x^2}\right)dx = \pi\left(\frac{4x^3}{3}+4x-\frac{1}{x}\right)\Big|_{1/2}^{1} =$
$\pi\left\{\frac{4}{3}+4-1-\frac{1}{6}-2+2\right\} = \frac{25\pi}{6}$.




3. $x^2+y^2 = 3^2 \Rightarrow 2x\frac{dx}{dt}+2y\frac{dy}{dt} = 0 \Rightarrow \frac{dx}{dt}\Big|_{x=1/2} = -\frac{y}{x}\frac{dy}{dt}\Big|_{x=1/2}$
$= -\frac{\sqrt{3^2-1/2^2}}{\frac{1}{2}}\left(-\frac{1}{4}\right) = \frac{1}{4}\sqrt{35}$ m/s

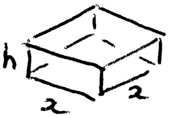
4. $g(x) = \cos(3x)-e^x+\ln(1+x) \Rightarrow g'(x) = -3\sin(3x)-e^x+\frac{1}{1+x} \Rightarrow g''(x) =$
$-9\cos(3x)-e^x-\frac{1}{(1+x)^2}$ so that $g(0) = \cos(0)-e^0+\ln(1) = 1-1+0 = 0$, $g'(0) =$
$-3\sin(0)-e^0+1 = 0$ and $g''(0) = -9\cos(0)-e^0-1 = -11$. Also, $h(x) =$
$\sin(x)+\ln(1-x) \Rightarrow h'(x) = \cos(x)-\frac{1}{1-x} \Rightarrow h''(x) = -\sin(x)-\frac{1}{(1-x)^2}$ so that
$h(0) = \sin(0)+\ln(1) = 0$, $h'(0) = \cos(0)-1 = 0$ and $h''(0) = -\sin(0)-1 = -1$

$\therefore \lim_{x\to 0}\frac{g(x)}{h(x)} = \frac{g''(0)}{h''(0)} = \frac{-11}{-1} = 11$

5(a) $f''(t) = -2t^{-3} \Rightarrow f'(t) = \int -2t^{-3}dt = t^{-2}+c$; $f'(1) = 3 \Rightarrow c = 2$.
Now $f(t) = \int(t^{-2}+2)dt = -t^{-1}+2t+b$ and $f(1) = 1 \Rightarrow b = 0$. So $f(t) = 2t-\frac{1}{t}$

(b) $u^2 = 4x+1 \Rightarrow x = \frac{u^2}{4}-\frac{1}{4} \Rightarrow \frac{dx}{du} = \frac{1}{2}u$. So $I = \int_{u=\sqrt{0+1}}^{u=\sqrt{8+1}} \frac{\frac{3(u^2-1)}{4}-1}{u}\,\frac{dx}{du}\,du =$
$\int_1^3 \frac{3u^2-7}{4u}\,\frac{u}{2}\,du = \frac{1}{8}\int_1^3(3u^2-7)du = \frac{u^3-7u}{8}\Big|_1^3 = \frac{27-21-1+7}{8} = \frac{3}{2}$

(c) $f(x) = |\sin(x)+4x| \Rightarrow f(-x) = |-\sin(x)-4x| = |\sin(x)+4x| = f(x) \Rightarrow f$ even.
So $I = 2\int_0^{\pi/2}f(x)dx = 2\int_0^{\pi/2}(\sin(x)+4x)dx = 2\left\{-\cos(x)+2x^2\right\}\Big|_0^{\pi/2} =$
$2\left\{-\cos(\pi/2)+\frac{\pi^2}{2}+\cos(0)-0\right\} = \pi^2+2$.

6. 


$4hx+x^2 = 48 \Rightarrow$
$h = \frac{12}{x}-\frac{x}{4} \Rightarrow$
Volume $= V = x^2h = 12x-\frac{x^3}{4}$
$\frac{dV}{dx} = 12-\frac{3x^2}{4}$; $\frac{d^2V}{dx^2} = -\frac{3x}{2} < 0$
$\Rightarrow$ max where $\frac{dV}{dx} = 0$ or $x = \sqrt{16} = 4$.
So largest possible volume is $48-\frac{64}{4} = 32\,m^3$

7. 

$x^2+y^2 = r^2$, $-r \le y \le 0$

$\delta V = \pi x^2\delta y + o(\delta y) = \pi(r^2-y^2)\delta y + o(\delta y)$
$\delta W = \rho\,\delta V g(|y|+\delta y) = \rho g\pi(r^2-y^2)|y|\delta y + o(\delta y)$
$\Rightarrow W = \int_{-r}^{0}\rho g\pi(r^2-y^2)|y|dy = \rho g\pi\int_{-r}^{0}(r^2-y^2)(-y)dy$
$= \rho g\pi\int_{-r}^{0}(-r^2y+y^3)dy = \rho g\pi\left(-\frac{r^2y^2}{2}+\frac{y^4}{4}\right)\Big|_{-r}^{0}$
$= \rho g\pi\left\{0+\frac{r^4}{2}-\frac{r^4}{4}\right\} = \frac{1}{4}\rho g\pi r^4$.