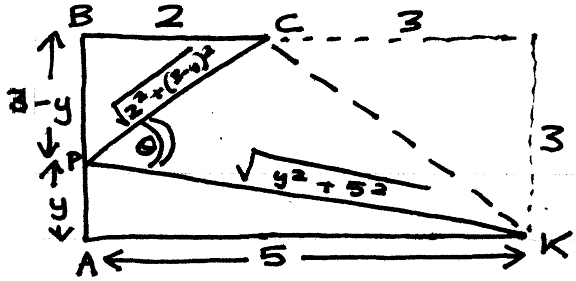

By the cosine rule, $CK^2 = PC^2 + PK^2 - 2\,PC\,PK\cos(\theta) \Rightarrow$

$$3^2+3^2 = 2^2+(3-y)^2+y^2+5^2 - 2\sqrt{4+(3-y)^2}\sqrt{25+y^2}\cos(\theta) \Rightarrow \cos(\theta) = u, \text{ where}$$

$$u = \frac{10-3y+y^2}{\sqrt{4+(3-y)^2}\sqrt{25+y^2}} \quad \text{(after simplification)}$$

Now, by the quotient rule, but also the product rule,

$$\frac{du}{dy} = \frac{\left\{(2y-3)\sqrt{4+(3-y)^2}\sqrt{5^2+y^2} - (10-3y+y^2)\times\left(\frac{1}{2}\{4+(3-y)^2\}^{-1/2}2(3-y)(-1)\sqrt{5^2+y^2} + \sqrt{4+(3-y)^2}\times\frac{1}{2}(5^2+y^2)^{-1/2}2y\right)\right.}{\text{denominator}^2}$$

$$= \frac{-9(5-y)(5-10y+y^2)}{(25+y^2)^{3/2}(13-6y+y^2)^{3/2}}$$

after much simplification. So for $0 < y < 3$ we have $\frac{du}{dy} = 0$ where $y^2-10y+5=0$ (because $5 \notin (0,3)$)

$$\Rightarrow y = \frac{10 \pm \sqrt{10^2-4.5}}{2}$$

$$= 5-\sqrt{20} \quad (\text{because } 5+\sqrt{20} \notin (0,3))$$

At $y = 5-\sqrt{20}$, $\frac{du}{dy}$ changes sign from $-$ to $+$

$\therefore$ u has a <u>minimum</u> where $y = 5-\sqrt{20}$

But θ is greatest where $\cos(\theta)$ is least. Therefore, the minimum of u corresponds to a maximum for θ, as required.