MAC 2311, Sections 09 & 10, Spring 2005 (Dr M-G)

Mock Second Test

Tuesday, March 1, 2005

You are allowed to use a TI-30Xa (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test. You may assume that

$$\frac{d}{du}\left\{\sinh^{-1}(u)\right\} = \frac{1}{\sqrt{u^2+1}}, \quad \frac{d}{du}\left\{\tan(u)\right\} = \frac{1}{\cos^2(u)}$$

(for $-\frac{\pi}{2} < u < \frac{\pi}{2}$ in the second case).

DO NOT WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT NOT STAPLED TO YOUR SOLUTIONS)

1. What is the tangent line to the curve $y = \dfrac{(2x^2+1)e^{x^2/2}}{2x+1}$ at the point $(1, \sqrt{e})$? [11]

2. Find both $m = \left.\dfrac{dy}{dx}\right|_{(x,y)=(-1,-1)}$ and $\alpha = \left.\dfrac{d^2y}{dx^2}\right|_{(x,y)=(-1,-1)}$ for $x^4 + x^2y^2 + y^5 = 1$ [11]

3. In each of the following cases, find $m = \left.\dfrac{dy}{dx}\right|_{x=0}$ *exactly*:

(a) $y = \ln\left(3 + 5x + 4x^2 + e^{2x+3x^2}\right)$ [3]

(b) $y = \ln\left(\sqrt{3x^2+4x+5}\right) + \sinh^{-1}(7x)$ [4]

4. A particle moves along the x-axis with position $x = \dfrac{\ln(2+t)+2t}{2+t}$ at time t.

(a) Find the particle's velocity at time t [3]

(b) Find the particle's acceleration at time t [3]

(c) Find the *precise* instant at which the velocity equals zero. [2]

(d) Find the *precise* instant at which the acceleration equals zero. [2]

5. Any related rates problem. One of many possibilities would be the following: A lighthouse is located on a small island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P? [6]

[Perfect score: $2 \times 11 + 7 + 10 + 6 = 45$]