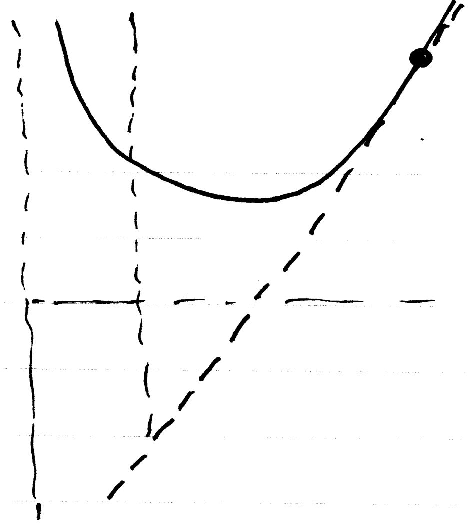
1. Using $\ln\left(\frac{A}{B}\right) = \ln(A) - \ln(B)$ and $\ln(AB) = \ln(A) + \ln(B)$ we have



$$\ln(y) = \ln(2x^2+1) + \ln\left(e^{x^2/2}\right) - \ln(2x+1)$$
$$= \ln(2x^2+1) + \tfrac{1}{2}x^2 - \ln(2x+1)$$

Now, using $\frac{d}{dx}[\ln(u)] = \frac{d}{du}[\ln(u)]\frac{du}{dx}$

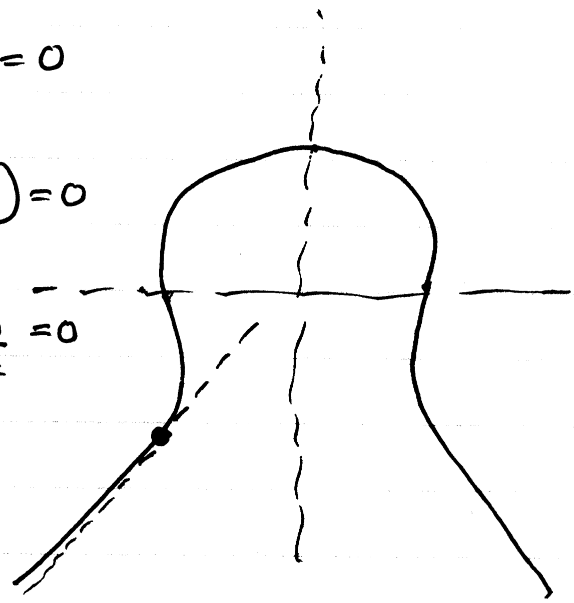
$$= \frac{1}{u}\frac{du}{dx}$$ we have

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2x^2+1}(2\cdot 2x+0) + \frac{1}{2}\cdot 2x - \frac{1}{2x+1}\cdot(2+0).$$

$\Rightarrow \frac{dy}{dx} = \left(\frac{4x}{2x^2+1} + x - \frac{2}{2x+1}\right)y$. Now set $x=1 \Rightarrow y = \sqrt{e}$ and $m = \left.\frac{dy}{dx}\right|_{x=1} = \left(\frac{4}{2+1} + 1 - \frac{2}{2+1}\right)\sqrt{e} = \frac{5}{3}\sqrt{e}$. So the tangent line has equation

$$y - \sqrt{e} = \frac{5}{3}\sqrt{e}(x-1) \quad \text{or} \quad y = \frac{1}{3}\sqrt{e}(5x-2)$$

2. $$\frac{d}{dx}\left\{x^4 + x^2y^2 + y^5\right\} = \frac{d}{dx}\{1\} = 0$$



$$\Rightarrow 4x^3 + \frac{d}{dx}(x^2)y^2 + x^2\frac{d}{dx}(y^2) + \frac{d}{dx}(y^5) = 0$$

(*) $$\Rightarrow 4x^3 + 2xy^2 + x^2\cdot 2y\frac{dy}{dx} + 5y^4\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{d}{dx}[4x^3] + 2\frac{d}{dx}[xy^2] + 2\frac{d}{dx}\left[x^2y\frac{dy}{dx}\right] + 5\frac{d}{dx}\left[y^4\frac{dy}{dx}\right] = 0$$

$$\Rightarrow 4\cdot 3x^2 + 2\left\{1\cdot y^2 + x\frac{d}{dx}(y^2)\right\} + 2\left\{2x\cdot y\cdot\frac{dy}{dx} + x^2\frac{d}{dx}(y)\frac{dy}{dx} + x^2y\frac{d}{dx}\left(\frac{dy}{dx}\right)\right\} + 5\left\{\frac{d}{dx}(y^4)\frac{dy}{dx} + y^4\frac{d}{dx}\left(\frac{dy}{dx}\right)\right\} = 0$$

(**) $$\Rightarrow 12x^2 + 2\left\{y^2 + x\cdot 2y\frac{dy}{dx}\right\} + 2\left\{2xy\frac{dy}{dx} + x^2\left(\frac{dy}{dx}\right)^2 + x^2y\frac{d^2y}{dx^2}\right\} + 5\left\{4y^3\frac{dy}{dx}\frac{dy}{dx} + y^4\frac{d^2y}{dx^2}\right\} = 0$$

Now put $x=-1$ and $y=-1$ in (*) and (**) to obtain

$$4(-1)^3 + 2(-1)(-1)^2 + (-1)^2 \cdot 2(-1)\,m + 5(-1)^4 m = 0 \quad \text{and}$$

$$12(-1)^2 + 2\left\{(-1)^2 + (-1)\,2(-1)\,m\right\} + 2\left\{2(-1)(-1)\,m + (-1)^2 m^2 + (-1)^2(-1)\,\alpha\right\}$$
$$+ 5\left\{4(-1)^3 m^2 + (-1)^4 \alpha\right\} = 0 .$$

or $\quad -4 - 2 - 2m + 5m = 0 \quad$ and

$$12 + 2(1 + 2m) + 2(2m + m^2 - \alpha) + 5(-4m^2 + \alpha) = 0$$

$$\Rightarrow \quad m = \frac{6}{3} = 2 \text{ and } \alpha = \frac{1}{3}\left\{18m^2 - 8m - 14\right\} = 14$$

3 (a) With $u = 3 + 5x + 4x^2 + e^{2x + 3x^2}$ we have

$$\frac{du}{dx} = 0 + 5 + 8x + e^{2x+3x^2}\frac{d}{dx}\left[2x + 3x^2\right]$$
$$= 5 + 8x + 2(1 + 3x)\,e^{2x+3x^2}$$

on using the chain rule (which yields $\frac{d}{dx}\left[e^w\right] = e^w \frac{dw}{dx}$).

Now, again using the chain rule, we have $y = \ln(u) \Rightarrow$

$$\frac{dy}{dx} = \frac{d}{du}\left[\ln(u)\right]\frac{du}{dx} = \frac{1}{u}\frac{du}{dx} \Rightarrow m = \left.\frac{dy}{dx}\right|_{x=0} =$$

$$\left.\frac{1}{u}\frac{du}{dx}\right|_{x=0} = \frac{1}{3 + 5\cdot 0 + 4\cdot 0^2 + e^0}\left(5 + 8\cdot 0 + 2(1 + 3\cdot 0)\,e^0\right)$$

$$= \frac{1}{3+1}\cdot(5+2) = \frac{7}{4}$$

(b) $$y = \ln\left[(3x^2 + 4x + 5)^{1/2}\right] + \sinh^{-1}(u)$$
$$= \frac{1}{2}\ln(3x^2 + 4x + 5) + \sinh^{-1}(u) \quad \text{with } u = 7x$$

So $$\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{3x^2+4x+5}\frac{d}{dx}\left\{3x^2 + 4x + 5\right\} + \frac{d}{du}\left[\sinh^{-1}(u)\right]\frac{du}{dx}$$

$$= \frac{1}{2}\,\frac{6x + 4 + 0}{3x^2 + 4x + 5} + \frac{1}{\sqrt{u^2+1}}\cdot 7$$

$$= \frac{3x + 2}{3x^2 + 4x + 5} + \frac{7}{\sqrt{49x^2 + 1}}$$

$$\Rightarrow \quad m = \left.\frac{dy}{dx}\right|_{x=0} = \frac{0+2}{0+0+5} + \frac{7}{\sqrt{0+1}} = \frac{37}{5}$$

4 (a) First note that $\frac{d}{dt}\left[\ln(u)\right] = \frac{1}{u}\frac{du}{dt} \Rightarrow \frac{d}{dt}\left[\ln(2+t)\right] = \frac{1}{2+t}(0+1)$

$$= \frac{1}{2+t} .$$

Now use the quotient rule to get

$$\text{velocity} = \frac{dx}{dt} = \frac{\frac{d}{dt}\{\ln(2+t)+2t\}(2+t) - (\ln(2+t)+2t)\frac{d}{dt}(2+t)}{(2+t)^2}$$

$$= \frac{\left(\frac{1}{2+t}+2\right)(2+t) - (\ln(2+t)+2t)(0+1)}{(2+t)^2}$$

$$= \frac{5 - \ln(2+t)}{(2+t)^2} \quad \text{and}$$

(b) $$\text{acceleration} = \frac{d^2x}{dt^2} = \frac{d}{dt}\left\{\frac{5-\ln(2+t)}{(2+t)^2}\right\}$$

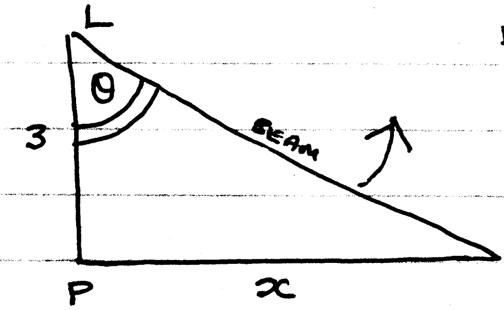
$$= \frac{\left(0 - \frac{1}{2+t}\right)(2+t)^2 - \{5-\ln(2+t)\}\,2(2+t)^1(0+1)}{(2+t)^4}$$

$$= \frac{(2+t)\{-1-10+2\ln(2+t)\}}{(2+t)^4} = \frac{2\ln(2+t)-11}{(2+t)^3}$$

(c) $$\frac{dx}{dt} = 0 \Rightarrow \ln(2+t) = 5 \Rightarrow t+2 = e^5 \Rightarrow t = e^5 - 2$$

(d) $$\frac{d^2x}{dt^2} = 0 \Rightarrow \ln(2+t) = \frac{11}{2} \Rightarrow t+2 = e^{11/2} \Rightarrow t = e^{11/2} - 2$$

5.



Four revs per min $= 4\cdot 2\pi = 8\pi$ rad/min

So $\frac{d\theta}{dt} = 8\pi$ rad/min

Also $\frac{x}{3} = \tan\theta \Rightarrow x = 3\tan(\theta)$

$$\text{So} \quad \frac{dx}{dt} = 3\frac{d}{dt}[\tan(\theta)] = 3\frac{d}{d\theta}[\tan(\theta)]\frac{d\theta}{dt}$$

$$= \frac{3}{\cos^2(\theta)}\frac{d\theta}{dt} = \frac{3}{\cos^2(\theta)}\cdot 8\pi = \frac{24\pi}{\cos^2(\theta)}$$

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$$\text{But} \quad x = 1 \Rightarrow \cos(\theta) = \frac{3}{\sqrt{3^2+1^2}} = \frac{3}{\sqrt{10}}$$

$$\text{So} \quad \left.\frac{dx}{dt}\right|_{x=1} = \frac{24\pi}{\left(\frac{3}{\sqrt{10}}\right)^2} = \frac{80\pi}{3} \text{ km/min}$$