

Mock Fourth Test
 Tuesday, April 19, 2005

You are allowed to use a TI-30Xa/TI-36X (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test. Note that

$$\frac{d}{d\xi} \{\arctan(\xi)\} = \frac{1}{\xi^2 + 1}.$$

DO **NOT** WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT **NOT** STAPLED TO YOUR SOLUTIONS)

1. (a) Calculate $\frac{d}{dx} \{\ln(\sqrt{x^2 + 1})\}$. [2] (b) Find the *exact* value of $\int_1^{\sqrt{3}} \frac{1 + 4x}{x^2 + 1} dx$. [4]

2. Given that $f(1) = 4$, $f'(1) = -10$ and

$$f''(t) = \frac{36 + 3t\sqrt{t} + 4t^3}{2t^3}$$

for all $t > 0$, find $f(t)$ *exactly*. [10]

3. Use the substitution $u = \sqrt[3]{21x + 1}$ to find the *exact* value of $I = \int_0^3 \frac{35x + 1}{\sqrt[3]{21x + 1}} dx$. [10]

4. Let R be the region bounded by $y = 16\sqrt[3]{x}$ and $x^3 = 16y$. Find the *exact* area of R

(a) by integrating with respect to x and [5]

(b) by integrating with respect to y . [5]

5. Let R be the region bounded by $y = 16\sqrt[3]{x}$ and $x^3 = 16y$. If R is rotated about the axis of symmetry $x = -1$, find the *exact* volume of the solid thus generated

(a) by integrating with respect to x and [6]

(b) by integrating with respect to y . [6]

6. Use L'Hôpital's rule to calculate $\lim_{x \rightarrow 0} f(x)$ where $f(x) = \frac{1}{x} - \frac{1}{2 \ln(1 + \frac{1}{2}x)}$. [12]