MAC 2311, Sections 09 & 10, Spring 2005 (Dr M-G)

Third Test

Tuesday, March 29, 2005

You are allowed to use a TI-30Xa/TI-36X (or any four-function calculator). No other calculator is allowed. You have 75 minutes. Present your solutions clearly. Show all necessary steps in your method. Include enough comments or diagrams to convince me that you thoroughly understand. Begin each question (as opposed to part of question) on a fresh sheet of paper, use *one* side of the paper only, and ensure that your solutions are stapled together in the proper order at the end of the test.

DO NOT WRITE ON THIS QUESTION PAPER, WHICH MUST BE TURNED IN AT THE END OF THE TEST (BUT NOT STAPLED TO YOUR SOLUTIONS)

1. A continuous function f is defined on $[0, 6]$ by

$$f(x) \;=\; \begin{cases} x^3 - 6x^2 + 9x + 2 & \text{if} \quad 0 \leq x < 4 \\ x^2 - 10x + 30 & \text{if} \quad 4 \leq x \leq 6. \end{cases}$$

(a) Find expressions for $f'(x)$ and $f''(x)$. [5]
(b) Find and identify all local extrema. [8]
(c) Where is f concave up? Where is f concave down? Find all inflection points. [3]
(d) Find both global extrema and all global extremizers of f on $[0, 6]$. [4]

2. Functions G and g are defined on $[0, 2]$ by $G(t) = \int\limits_0^t g(x)\,dx$ and

$$g(t) \;=\; \begin{cases} 3t & \text{if} \quad 0 \leq t < 1 \\ 2t^2 + t^3 & \text{if} \quad 1 \leq t \leq 2. \end{cases}$$

(a) Verify that g is continuous on $[0, 2]$ [2]
(b) Assuming that $\int_a^b x^s\,dx = \frac{b^{s+1} - a^{s+1}}{s+1}$ for $0 \leq s \leq 3$ but *without invoking the fundamental theorem of calculus*, find an explicit formula for $G(t)$ for all $t \in [0, 2]$. [6]

3. In each of the following cases, find the exact value of the definite integral:

(a) $I = \int\limits_1^2 \left\{x - \dfrac{1}{x}\right\}^3 dx$ [10]

(b) $I = \int\limits_1^4 (3 - 2\sqrt{x})(5 - 2\sqrt{x})\,dx$ [10]

4. Find $F'(t)$ for $F(t) = \int\limits_1^{e^{3t}} \ln(x)\,dx$, simplifying your answer as much as possible. [6]

5. **(a)** An open-topped box in the shape of a right-circular cylinder must have a volume of 8π cm^3. Find the radius and height that minimize the surface area of this box. [5]
(b) What is the resulting surface area? [1]

[Perfect score: $20 + 8 + 20 + 2 \times 6 = 60$]