1 (a) $f'(x) = \begin{cases} 3x^2-12x+9 & \text{if } x\in(0,4) \\ 2x-10 & \text{if } x\in(4,6) \end{cases} = \begin{cases} 3(x-1)(x-3) & \text{if } x\in(0,4) \\ 2(x-5) & \text{if } x\in(4,6) \end{cases}$

$f''(x) = \begin{cases} 6x-12 & \text{if } x\in(0,4) \\ 2 & \text{if } x\in(4,6) \end{cases} = \begin{cases} 6(x-2) & \text{if } x\in(0,4) \\ 2 & \text{if } x\in(4,6) \end{cases}$

(b) Only possibility for a corner extremizer is $x=4$. We have $f'(4-) = 3(4-1)(4-3) = 9 > 0$ and $f(4+) = 2\cdot4-10 = -2 < 0$. So $f(4-) > 0 > f(4+) \Rightarrow$ corner local maximum. All other local extrema must be smooth. Hence $f'(x)=0 \Rightarrow x=1, x=3$ or $x=5$. $f''(1) = -6 < 0$, $f''(3) = 6 > 0$ and $f''(5) = 2 > 0$. So $x=1$ is a local maximizer and $x=3, x=5$ are local minimizers.

(c) $f''(4-) = 6(4-2) = 12 > 0$, $f''(4+) = 2 > 0 \Rightarrow f''$ does not change sign at $x=4$. So the only inflection point is at $x=2$, where f'' changes sign from $-$ to $+$. Hence concave down on $(0,2)$ and concave up on $(2,4)\cup(4,6)$.

(d) The candidates for global extremizer are the endpoints $x=0, x=6$ and the critical points $x=1$, $x=3$, $x=4$ and $x=5$. We have $f(0)=2$, $f(1)=6$, $f(3)=2$, $f(4)=6$, $f(5)=5$ and $f(6)=6$. So the global maximum is 6 and the global minimum is 2. There are three global maximizers, namely, 1, 4 and 6; and two global minimizers, namely, 0 and 3.

2 (a) $g(1^-) = 3\cdot1 = 3$; $g(1^+) = 2\cdot1^2+1^3 = 3$. So $g(1^-)=g(1^+) \Rightarrow g$ continuous at $t=1$. $\therefore$ g is continuous everywhere, being piecewise polynomial.

(b) For $0\le t\le 1$, $G(t) = \int_0^t g(x)\,dx = \int_0^t 3x\,dx = 3\int_0^t x\,dx$ $= 3\left(\frac{t^2-0^2}{2}\right) = \frac{3}{2}t^2$. For $1\le t\le 2$, $G(t) = \int_0^t g(x)\,dx$

$= \int_0^1 g(x)\,dx + \int_1^t g(x)\,dx = G(1) + \int_1^t (2x^2+x^3)\,dx =$

$\frac{3}{2}1^2 + 2\int_1^t x^2\,dx + \int_1^t x^3\,dx = \frac{3}{2} + 2\left(\frac{t^3-1^3}{3}\right) + \frac{t^4-1^4}{4}$.

So $G(t) = \begin{cases} \frac{3}{2}t^2 & \text{if } 0\le t\le 1 \\ \frac{7}{12}+\frac{2}{3}t^3+\frac{1}{4}t^4 & \text{if } 1\le t\le 2 \end{cases}$

3 (a) $I = \int_1^2 \left(x - \frac{1}{x}\right)^3 dx = \int_1^2 \left\{ x^3 + 3x^2\left(\frac{-1}{x}\right) + 3x\left(\frac{-1}{x}\right)^2 + \left(\frac{-1}{x}\right)^3 \right\} dx$

$= \int_1^2 x^3\,dx - 3\int_1^2 x\,dx + 3\int_1^2 \frac{1}{x}\,dx - \int_1^2 x^{-3}\,dx$

$= \int_1^2 \frac{d}{dx}\left(\frac{x^4}{4}\right)dx - 3\int_1^2 \frac{d}{dx}\left(\frac{x^2}{2}\right)dx + 3\int_1^2 \frac{d}{dx}\{\ln(x)\}dx - \int_1^2 \frac{d}{dx}\left(-\frac{1}{2}x^{-2}\right)dx$

$= \left.\frac{x^4}{4}\right|_1^2 - 3\left.\frac{x^2}{2}\right|_1^2 + 3\ln(x)\Big|_1^2 - \left(-\frac{1}{2}x^{-2}\right)\Big|_1^2$

$= \frac{1}{4}(2^4 - 1^4) - \frac{3}{2}(2^2 - 1^2) + 3(\ln(2) - \ln(1)) - \left\{-\frac{1}{2}2^{-2} - \left(-\frac{1}{2}1^{-2}\right)\right\}$

$= 4 - \frac{1}{4} - 6 + \frac{3}{2} + 3\ln(2) - 0 + \frac{1}{8} - \frac{1}{2} = 3\ln(2) - \frac{9}{8}$

(b) $I = \int_1^4 (3 - 2\sqrt{x})(5 - 2\sqrt{x})\,dx = \int_1^4 (15 - 16x^{1/2} + 4x)\,dx$

$= \int_1^4 \frac{d}{dx}\left\{15x - \frac{32}{3}x^{3/2} + 2x^2\right\}dx = \left(15x - \frac{32}{3}x^{3/2} + 2x^2\right)\Big|_1^4$

$= 15\cdot 4 - \frac{32}{3}4\sqrt{4} + 2\cdot 4^2 - \left(15 - \frac{32}{3} + 2\right)$

$= 60 - \frac{256}{3} + 32 - 15 + \frac{32}{3} - 2 = \frac{1}{3}$

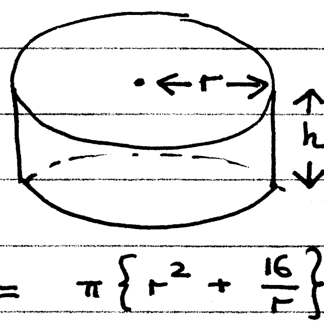
4 Put $u = e^{3t} \Rightarrow \frac{du}{dt} = 3e^{3t}$. Then if $y = \int_1^u \ln(x)\,dx$

we have $F'(t) = \frac{dy}{dt} = \frac{dy}{du}\frac{du}{dt} = \frac{d}{du}\left\{\int_1^u \ln(x)\,dx\right\} 3e^{3t}$

$= \ln(u)\,3e^{3t} = \ln(e^{3t})\cdot 3e^{3t} = 3t\cdot 3e^{3t}$

$= 9te^{3t}$

5.

(a) 

Volume $= \pi r^2 h$. So $\pi r^2 h = 8\pi \Rightarrow$

$h = 8/r^2$. Surface area is

$S = \pi r^2 + 2\pi r.h = \pi\{r^2 + 2rh\}$

$= \pi\left\{r^2 + \frac{16}{r}\right\} = \pi(r^2 + 16r^{-1}) \Rightarrow \frac{dS}{dr} = \pi\{2r - 16r^{-2}\} \Rightarrow$

$\frac{d^2S}{dr^2} = \pi\{2 + 32r^{-3}\} > 0$. So unique global min where $\frac{dS}{dr} = 0$

$\Rightarrow 2r = \frac{16}{r^2} \Rightarrow r^3 = 8 \Rightarrow r = 2$. Then $h = \frac{8}{2^2} = 2$

(b) $\pi\{r^2 + 2rh\} = \pi\{2^2 + 2\cdot 2\cdot 2\} = 12\pi$ cm^2.