1 (a) $\int_0^4 |x^2-4|\,dx = \int_0^2 (4-x^2)\,dx + \int_2^4 (x^2-4)\,dx = \left\{4x - \frac{1}{3}x^3\right\}\Big|_0^2$
$+ \left\{\frac{1}{3}x^3 - 4x\right\}\Big|_2^4 = 8 - \frac{8}{3} + \frac{64}{3} - 16 - \left(\frac{8}{3} - 8\right) = 16$

(b) Put $u = t^{1/2}$. Then $y = \int_1^u \sin(x^2)\,dx \Rightarrow F(t) = y$ and
$\frac{dy}{du} = \sin(u^2)$. So $F'(t) = \frac{dy}{dt} = \frac{dy}{du}\frac{du}{dt} = \sin(u^2)\cdot\frac{1}{2}t^{-1/2}$
$= \frac{\sin(t)}{2\sqrt{t}}$

2 $f'(t) = \int \frac{1}{t^2}\,dt = \int t^{-2}\,dt = -t^{-1} + c$; $f'(1) = 2 \Rightarrow c = 3$
Now $f'(t) = 3 - \frac{1}{t} \Rightarrow f(t) = \int\left(3 - \frac{1}{t}\right)dt = 3t - \ln(t) + b$;
$f(1) = 3 \Rightarrow b = 3 - 3 + \ln(1) = 0$. So $f(t) = 3t - \ln(t)$

3 $\frac{dy}{dx} = (2\cdot 1 + 0)e^{3x} + (2x+5)\cdot 3e^{3x} \Rightarrow \frac{dy}{dx}\Big|_{x=0} = 2e^0 + 15e^0$
$= 17$. So tangent is $y - 5 = 17(x-0)$ or $y = 17x + 5$

4. $\frac{dx}{dt} = \dfrac{\frac{d}{dt}\{\ln(1+2t)\}(1+2t) - \ln(1+2t)\cdot(0+2)}{(1+2t)^2}$ and $\frac{d}{dt}\{\ln(1+2t)\}$
$= \frac{1}{1+2t}\cdot(0+2) \Rightarrow \frac{dx}{dt} = 2\{1 - \ln(2t+1)\}\cdot(1+2t)^{-2}$. So
velocity $= 0 \Rightarrow \frac{dx}{dt} = 0 \Rightarrow \ln(1+2t) = 1 \Rightarrow 1 + 2t = e \Rightarrow t = \frac{1}{2}(e-1)$

5. $u = \sqrt{x-1} \Rightarrow x = 1 + u^2 \Rightarrow \frac{dx}{du} = 0 + 2u$. So $I =$
$\int_{u=\sqrt{2-1}}^{u=\sqrt{4-1}} \frac{1}{x\sqrt{x-1}}\frac{dx}{du}\,du = \int_1^{\sqrt{3}} \frac{1}{(u^2+1)\,u} 2u\,du = 2\int_1^{\sqrt{3}} \frac{1}{1+u^2}\,du$
$= 2\arctan(u)\Big|_1^{\sqrt{3}} = 2\{\arctan(\sqrt{3}) - \arctan(1)\} = 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\pi}{6}$

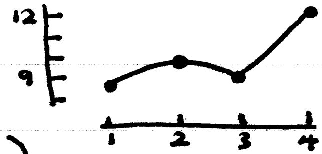
6(a) $f(3-) = 11 + \ln\left(\frac{27}{3^3}\right) - \frac{6}{3} = 11 + \ln(1) - 2 = 9$
$f(3+) = 3\cdot 3 = 9$. So $f(3-) = f(3+) \Rightarrow f$ is continuous,
because $\ln$ and power functions are continuous on $(0, \infty)$

(b) $f(x) = 11 + \ln(27) - \ln(x^3) - 6x^{-1} = 11 + \ln(27) - 3\ln(x) - 6x^{-1}$
if $1 \leq x < 3$. Hence $f'(x) = \begin{cases} -\frac{3}{x} + \frac{6}{x^2} & \text{if } 1 < x < 3 \\ 3 & \text{if } 3 < x < 4 \end{cases}$

Because $f'(3-) = \frac{-3}{3} + \frac{6}{3^2} = -1/3 < 0$ and $f'(3+) = 3 > 0$,
there is a corner local min at $x = 3$. Any other local extremum must
be smooth. For $x \in (3,4)$, $f'(x) = 3 \neq 0$. For $x \in (1,3)$, $f'(x)$
$= \frac{6-3x}{x^2} = \frac{3(2-x)}{x^2} \Rightarrow f'(2) = 0$. Also, $f''(x) = \frac{3}{x^2} - \frac{12}{x^3}$
on $(1,3) \Rightarrow f''(2) = \frac{3}{4} - \frac{12}{8} = -\frac{3}{4} < 0$. So there's a local

max at $x=2$.

(c) The candidates for global extremizer are $x = 1$ (endpoint), $x = 2$, $x = 3$ and $x = 4$. We have $f(1) = 5 + \ln(27)$, $f(2) = 8 + 3\ln\left(\frac{3}{2}\right)$, $f(3) = 9$ and $f(4) = 12$. The biggest number is 12, which is therefore the global max; the smallest is $5 + \ln(27)$, which is therefore the global min (must have $5 + \ln(27) < 9$ because $\ln(27) < 4$)



7 (a) $\displaystyle\int_1^2 \left(x + \frac{2}{x} - 1\right) dx = \left(\frac{1}{2}x^2 + 2\ln(x) - x\right)\Big|_1^2 = 2 + 2\ln(2) - 2 - \frac{1}{2} - 2\ln(1) + 1$

$$= \frac{1}{2} + 2\ln(2)$$

(b) Use shells. Get $\displaystyle\int_1^2 2\pi x\left(x + \frac{2}{x} - 1\right) dx = 2\pi\int_1^2 (x^2 + 2 - x)\, dx$

$$= 2\pi\left(\frac{1}{3}x^3 + 2x - \frac{1}{2}x^2\right)\Big|_1^2 = 2\pi\left\{\frac{8}{3} + 4 - 2 - \frac{1}{3} - 2 + \frac{1}{2}\right\} = \frac{17\pi}{3}$$

(c) Use disks. Get $\displaystyle\int_1^2 \pi\left(x + \frac{2}{x} - 1\right)^2 dx = \pi\int_1^2 \left\{x^2 + \frac{4}{x^2} + 1 + 2x\cdot\frac{2}{x}\right.$

$$\left. + 2x(-1) + 2\cdot\frac{2}{x}(-1)\right\} dx = \pi\int_1^2 \left(x^2 - 2x + 5 - \frac{4}{x} + 4x^{-2}\right) dx$$

$$= \pi\left\{\frac{1}{3}x^3 - x^2 + 5x - 4\ln(x) - 4x^{-1}\right\}\Big|_1^2 = \pi\left\{\frac{8}{3} - 4 + 10 - 4\ln(2)\right.$$

$$\left. - 2\right\} - \pi\left\{\frac{1}{3} - 1 + 5 - 4\ln(1) - 4\right\} = \frac{1}{3}\left(19 - 12\ln(2)\right)\pi$$

8. Set $g(x) = \ln(1+x) - \arctan(x) \Rightarrow g'(x) = \dfrac{1}{1+x} - \dfrac{1}{1+x^2} \Rightarrow$

$$g''(x) = -(1+x)^{-2} + (1+x^2)^{-2}\cdot(0+2x) = \frac{2x}{(1+x^2)^2} - \frac{1}{(1+x)^2}$$

and $h(x) = \cos(2x) - e^x + \sin(x) \Rightarrow h'(x) = -2\sin(2x) - e^x + \cos(x) \Rightarrow h''(x) = -2^2\cos(2x) - e^x - \sin(x)$

So we have $g(0) = 1 - 0 = 0$, $g'(0) = 1 - 1 = 0$ and $g''(0) = 0 - 1 = -1$; and $h(0) = 1 - 1 + 0 = 0$, $h'(0) = -0 - 1 + 1 = 0$ and $h''(0) = -4 - 1 - 0 = -5$. So, by L'Hôpital's rule,

$$\lim_{x\to 0}\frac{g(x)}{h(x)} = \lim_{x\to 0}\frac{g''(x)}{h''(x)} = \frac{g''(0)}{h''(0)} = \frac{-1}{-5} = \frac{1}{5}$$