

Fourth Assignment

Due in ink at 1:25 p.m. on Wednesday, April 26, 2017

1. Use optimal control theory to show that the shortest path between any two points in a plane is a straight line. [10]

2. Solve the problem of time-optimal control to the origin for

$$\dot{x}_1 = x_1 + 2x_2, \quad \dot{x}_2 = 4x_1 - x_2 + u,$$

where $|u| \leq 1$. Identify the region from which the system is controllable. [15]

3. Solve the problem of time-optimal control to the origin for

$$\dot{x}_1 = e^{x_2}, \quad \dot{x}_2 = u,$$

where $|u| \leq 1$. Identify the region $\mathfrak{S} \subset \mathbb{R}^2$ from which the system is controllable, and find x^* and t_1^* for $x^0 \in \mathfrak{S}$. [15]