First Assignment

Due at 1:25 p.m. on Friday, February 10, 2017

1. Find an admissible extremal for the problem of minimizing

$$J[x] \;=\; \int_0^{\frac{\pi}{2}} \{x^2 - \dot{x}^2 - 2x\sin(t)\}\,dt$$

subject to $x(0) = 0$ and $x\left(\frac{\pi}{2}\right) = 1$. [10]

2. **(a)** Show that there is no admissible extremal for the problem of minimizing

$$J[y] \;=\; \int_0^2 y^2(1 - y')^2\,dx$$

subject to $y(0) = 0$ and $y(2) = 1$.

(b) Find by inspection a broken extremal that minimizes $J[y]$. [10]

3. For the problem of minimizing

$$J[x] \;=\; \int_0^{\sqrt{2}} \{\dot{x}^2 + 2tx\dot{x} + t^2x^2\}\,dt$$

subject to $x(0) = 1$ and $x(\sqrt{2}) = 1/e$:

(a) Show that $\phi(t) = e^{-t^2/2}$ is an admissible extremal.

(b) Use a direct method to confirm that ϕ is the minimizer. [10]

4. Find an admissible extremal for the problem of minimizing

$$J[x] \;=\; \int_1^2 \frac{\sqrt{1 + (\dot{x})^2}}{x}\,dt$$

with $x(1) = 2$ and $x(2) = 1$. [10]

Hint: Use the substitution $\dot{x} = \tan(\theta)$.

5. Show that the admissible extremal for

$$J[y] \;=\; \int_0^1 \cos^2(y')\,dx$$

with $y(0) = 0$ and $y(1) = 1$ is not the minimizer over either **(a)** D_1 or **(b)** C_1. [10]

Hint: It may help to read over the first remark on p. 25 of the text again.

[Perfect score: $5 \times 10 = 50$]