Fourth Assignment

Due in ink at 1:25 p.m. on Wednesday, April 26, 2017

1. Use optimal control theory to show that the shortest path between any two points in a plane is a straight line. [10]

2. Solve the problem of time-optimal control to the origin for

$$\dot{x}_1 = x_1 + 2x_2, \qquad \dot{x}_2 = 4x_1 - x_2 + u,$$

where $|u| \leq 1$. Identify the region from which the system is controllable. [10]

3. Solve the problem of time-optimal control to the origin for

$$\dot{x}_1 = e^{x_2}, \qquad \dot{x}_2 = u,$$

where $|u| \leq 1$. Identify the region $\mathfrak{S} \subset \Re^2$ from which the system is controllable, and find x^* and t_1^* for $x^0 \in \mathfrak{S}$. [15]

4. The system

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = u$$

subject to $|u| \leq 1$ is to be controlled from $x(0) = (0, 1)$ to $x(t_1) = (0, \beta)$ in such a way as to minimize

$$J = \tfrac{1}{2} \int_0^{t_1} \{x_2^2 - x_1^2\}\, dt$$

for suitable t_1. The optimal trajectory is a concatenation of arcs from three different phase-planes.

(a) Describe each phase-plane.

(b) If it is known that there is no control switch for $\beta = -1$, what must be the optimal control?

(c) If it is known that there is precisely one control switch for $\beta = -\sqrt{2}$, what must be the optimal control sequence? [15]

[Perfect score: $2 \times 10 + 2 \times 15 = 50$]