5 Richardsonian Models of War Psychology

The Lanchestrian models of Lecture 2 concern themselves only with the course of a war. They do not address the arms race or crisis that may have precipitated that war. Such models were pioneered by Richardson (1960). According to Rashevsky and Trucco (1960, p. ix), the value of Richardson's work "lies not in the particular formulation of his theory but in the fact that Richardson shows *how* the problems of the causes of war can be subject to mathematical treatment and to rigorous mathematical thought ... Whatever the shortcomings of this book, it will have to be studied by every investigator who delves into the causes and origins of war ... " Needless to say, that includes us!

Richardson's most basic assumption is that feelings of fear, rivalry, grudges, etc., between nations result in armaments races. His fundamental equations "determine the armaments races of two nations in terms of various psychological, sociological, and economic factors" (Rashevsky and Trucco, 1960, p. vi). These equations are

$$\frac{dx}{dt} = ky - \alpha x + g \tag{5.1a}$$
$$\frac{dy}{dt} = lx - \beta y + h \tag{5.1b}$$

where x and y denote the armaments levels of Nations 1 and 2, respectively, at time t. Here k and l are positive "defense coefficients" (with dimensions TIME^{-1}); α and β are "positive constants representing the fatigue and expense of keeping up defenses" (likewise with dimensions TIME^{-1}); and g and h represent the grievances felt towards the other side. Each "is a positive number when its side is dissatisfied and a negative number when the prevailing mood of that side is contentment" (Richardson, 1960, pp. 15–16).

If g, h, x and y are all made zero simultaneously, then they remain zero. Richardson (1960, pp. 16–17) calls this ideal condition "permanent peace by disarmament and satisfaction" and notes that "mutual disarmament without satisfaction is not permanent, for, if x and y instantaneously vanish, $\frac{dx}{dt} = g$ and $\frac{dy}{dt} = h$" are both positive. The equations likewise imply that unilateral disarmament is not permament: if, say, $y = 0$ at any time, then it will not remain zero unless x and h are also zero.

Let us define

$$x_0 = \frac{\beta g + kh}{\alpha\beta - kl}, \qquad y_0 = \frac{lg + \alpha h}{\alpha\beta - kl} \tag{5.2}$$

and assume $\alpha\beta \neq kl$. By standard analysis, (5.1) has a unique equilibrium at $(x, y) = (x_0, y_0)$, and this equilibrium is stable for

$$\alpha\beta > kl \tag{5.3}$$

but unstable for[1]

$$\alpha\beta < kl. \tag{5.4}$$

[1] Because setting $X = x - x_0$, $Y = y - y_0$ yields $\frac{dX}{dt} = kY - \alpha X$ and $\frac{dY}{dt} = lX - \beta Y$, from which Y is readily eliminated to yield $\frac{d^2X}{dt^2} + (\alpha+\beta)\frac{dX}{dt} + (\alpha\beta - kl)X = 0$, whose characteristic equation $\lambda^2 + (\alpha+\beta)\lambda + \alpha\beta - kl = 0$ invariably has two real roots, namely, $\frac{1}{2}\{-\alpha - \beta \pm \sqrt{(\alpha-\beta)^2 + 4kl}\} = \frac{1}{2}\{-|\alpha+\beta| \pm \sqrt{|\alpha+\beta|^2 - 4(\alpha\beta - kl)}\}$.

Stable equilibrium can be interpreted as a balance of power with a constant positive level of expenditure on each side, but unstable equilibrium arises for negative x_0 and y_0 whenever g and h are both positive. How can the level of armaments be negative?

Before answering that question, let us consider the possibility that (5.4) holds with g and h both negative. Then x_0 and y_0 are both positive and both sides are satisfied—have no aggressive intent—yet when the equilibrium is unstable, an arms race will still result. Since it is so hard to be sure whether g and h are positive or negative, this case highlights the "tragic moral dilemma" that led Richardson (1960, pp. 27–28) to say, "Personally, I think that much of what is blamed as aggressive intentions (g or h) is really only defensiveness (k or l).

Returning now to the question of how x and y can be negative, Richardson (1960, p. 32) reinterpreted x and y as differences between positive contributions to threatening another nation and positive contributions towards cooperating with it by writing

$$x \;=\; U - U_0, \qquad y = V - V_0 \tag{5.5}$$

where, in essence, U stands for Nation 1's defense budget, V stands for Nation 2's defense budget, U_0 stands for exports from Nation 1 to Nation 2 and V_0 stands for exports from Nation 2 to Nation 1, all measured in terms of the relevant unit of currency (which for Richardson was £ sterling). So x and y will both be negative when $U < U_0$ and $V < V_0$.

With this reinterpretation, Richardson was able to test his equations against the European arms race that preceded World War I, with the alliance between France and Russia as Nation 1 and the alliance between Germany and Austria-Hungary as Nation 2. Because the opposing alliances were similar in size, Richardson assumed for simplicity that both $k = l$ and $\alpha = \beta$, so that adding (5.1a) and (5.1b) yields

$$\frac{d(x+y)}{dt} \;=\; (k-\alpha)(x+y) + g + h. \tag{5.6}$$

He also assumed for simplicity that U_0 and V_0 are both constant. Then (5.5)–(5.6) imply

$$\frac{d(U+V)}{dt} \;=\; (k-\alpha)\left\{U + V - \left(U_0 + V_0 - \frac{g+h}{k-\alpha}\right)\right\} \tag{5.7}$$

and hence that plotting $\frac{d}{dt}(U+V)$ against $U+V$ should yield points that lie on a line (if the model is a good approximation of reality). Richardson used the data in Table 5.1 together with the midpoint rule to obtain four data points for such a plot, namely, $(202, 5.6)$, $(209.85, 10.1)$, $(226.8, 23.8)$ and $(263.85, 50.3)$.[2] It can be seen from Figure 5.1 that there is, in Richardson's words, a "marvelously good fit" between the best-fit line and the data points. The slope of the line is about 0.73 (YEAR^{-1}), and it cuts the horizontal axis where $U + V \approx 195$; we may therefore estimate that

$$k - \alpha \;\approx\; 0.73 \tag{5.8}$$

and $U_0 + V_0 - (g+h)/(k-\alpha) \;\approx\; 195$. Note that, with $\alpha = \beta$ and $k = l$, (5.4) makes (5.2) an unstable equilibrium of (5.1) if $(k-\alpha)(k+\alpha) > 0$, which (5.8) clearly implies.

[2]For example, at the mid point between 1909 and 1910, $\frac{d(U+V)}{dt}$ is estimated as $\frac{204.8-199.2}{1} = 5.6$ and $U+V$ is estimated as $\frac{1}{2}\{204.8 + 199.2\} = 202$.

Country	1909	1910	1911	1912	1913
France	48.6	50.9	57.1	63.2	74.7
Russia	66.7	68.5	70.7	81.8	92.0
Germany	63.1	62.0	62.5	68.2	95.4
Austria	20.8	23.4	24.6	25.5	26.9
Total	199.2	204.8	214.9	238.7	289.0

Table 5.1: Defense budgets in $\pounds 10^6$ of some European nations (Richardson, 1960, p. 32)

Richardson interprets the graph as indicating that "the good will between the opposing alliances" would just have covered $\pounds 195,000,000$ of defense expenditure on the part of the four nations concerned. But their actual expenditure in 1909 was $\pounds 199$ millions; "and so began an arms race which led to World War I" (Richardson, 1960, pp. 33–34).

Nevertheless, a nation may react less to its opponent's level of armaments than to the difference between that level and its own, in which case, (5.1) should be replaced by

$$\frac{dx}{dt} = K(y-x) - \alpha x + g = Ky - (K+\alpha)x + g \tag{5.9a}$$

$$\frac{dy}{dt} = L(x-y) - \beta y + h = Lx - (L+\beta)y + h \tag{5.9b}$$

where $K, L > 0$. Richardson regards (5.9) as driven by rivalry, and (5.1) as driven by fear or apprehension. Comparison with (5.1) shows that the two dynamical systems become identical when k, l, α and β are replaced by K, L, $\alpha + K$ and $\beta + L$, respectively. Hence, by (5.3), the system is stable if $(\alpha + K)(\beta + L) > KL$ or $\alpha\beta + \beta K + \alpha L > 0$, which always holds, and so (5.9) is "not an adequate description of the interaction between nations" because "arms races do occur"—but Richardson also has "no doubt" that rivalry "has some effect" (Richardson, 1960, p. 36), even though (5.9) fails to capture it adequately.

These models are only a beginning. They have been further developed by Richardson himself and by numerous others since. If any of you is sufficiently interested, perhaps this work could form the basis of a more in-depth investigation leading to an end-of-term presentation.

For the rest of us, however, this is as far as we go with it for now!

Figure 5.1: Fit to (5.7) of European arms race prior to World War I (Richardson, 1960, p. 33)