Section 2: Loan Repayment – Sinking Fund Method

IDEA: Each period, pay interest only to the lender, and in a separate account (the sinking fund account), make payments that will *accumulate* to the loan balance. The interest rate charged by the lender is generally different than the interest rate used for the sinking fund account.

The total amount paid at time k is denoted by R_k. This amount can be separated into two parts, the amount of the interest payment to the lender, denoted by R_k^I, and the amount of the payment made into the sinking fund, denoted by R_k^{SF}. So we have

$$R_k = R_k^I + R_k^{SF}$$

Since the sinking fund deposits accumulate to the loan balance, we generally have $L = AV(R_k^{SF})$.

Remarks:

1. If the sinking fund interest rate and the loan interest rate are equal, then the sinking fund and amortization methods are equivalent. (One is indifferent to the two methods.)

2. The net amount of interest paid on the loan during a certain installment (period) equals the difference between the amount of interest paid to the lender and the amount of interest earned in the sinking fund.

Basic Relationships for Level Payment Sinking Funds:
Level payments imply $R_k^{SF} = R^{SF}$ for all k; that is, the payments are constant.
i = periodic effective interest rate on the loan, charged by the lender
j = periodic effective interest rate used for the sinking fund

$$R^I = L \cdot i$$

$$L = R^{SF} s_{\overline{n}|j} \quad \Longrightarrow \quad R^{SF} = \frac{L}{s_{\overline{n}|j}}$$

Then the total periodic payment is $R = L \cdot i + \frac{L}{s_{\overline{n}|j}} = L(i + \frac{1}{s_{\overline{n}|j}})$. Rewriting, we have

$$L = \frac{R}{i + \frac{1}{s_{\overline{n}|j}}}$$

Module 3 Section 2 Problems:

1. A 20-year loan of 100,000 is to be repaid with annual payments using a sinking fund. The lender charges 6% per annum and interest is credited in the sinking fund at 4% per annum.

   (a) Determine the amount of the interest payment to the lender each year.
   (b) Determine the amount of the sinking fund payment each year.
   (c) Determine the total amount of each annual payment.
   (d) Determine the net amount of interest paid during the 13th installment.

2. A 10-year loan of 40,000 is to be repaid with monthly payments using the sinking fund method in which interest on the principal is determined using 4% annual effective and deposits into the sinking fund also earn 4% annual effective. Determine the total amount of each monthly payment.

3. A 30-year mortgage of 500,000 is to be repaid with quarterly payments using the sinking fund method in which interest on the principal is determined using 8% compounded quarterly and deposits into the sinking fund earn 6% compounded quarterly. The first quarterly deposit into the sinking fund is X and each subsequent deposit is 25 more than its preceding deposit. Determine the amount of the 45th payment.

4. A 15-year loan is repaid using the sinking fund method with annual payments to the lender determined using 5% annual effective, and non-level annual payments into the sinking fund that are credited using 4.03% annual effective. The initial sinking fund payment is 1000, and each subsequent payment into the sinking fund is 3% more than its preceding payment. Determine the net amount of interest paid in the 12th payment.

Answers to Module 3 Section 2 Problems

1) (a) 6000

(b) 3358.18

(c) 9358.18

(d) 3981.63

2) 403.62

3) 11546.30

4) 590.94