

MAP 4170 - 01
Test 1

Name: Key
Date: September 10, 2007

Each problem below is worth 10 points. Use correct notation.

1. Calculate the nominal rate of interest compounded quarterly that is equivalent to a nominal rate of discount of 18.9% compounded monthly.

A. 18.6% B. 18.9% C. 19.2% D. 19.5% E. 19.8%

Given $d^{(12)} = .189$. Find $i^{(4)}$.

$$\left. \begin{array}{l} \xrightarrow{d^{(12)} = .189} (1 - \frac{.189}{12})^{-12} \\ \xrightarrow{i^{(4)}} (1 + \frac{i^{(4)}}{4})^4 \end{array} \right\} \Rightarrow (1 - \frac{.189}{12})^{-12} = (1 + \frac{i^{(4)}}{4})^4$$

$$\Rightarrow (1 - \frac{.189}{12})^{-3} = 1 + \frac{i^{(4)}}{4}$$

$$\Rightarrow i^{(4)} = 19.5\%$$

2. The sum $v + v^2 + v^3 + \dots + v^n$ simplifies to which of the following expressions?

A. $\frac{v-v^n}{1-v}$ B. $\frac{1-v^{n+1}}{1-v}$ C. $\frac{1-v^n}{v}$ D. $\frac{1-v^n}{d}$ E. $\frac{1-v^n}{i}$

$$\sum = \frac{v - v^{n+1}}{1 - v} = \frac{v(1 - v^n)}{1 - v}$$

$$\frac{v}{1-v} = \frac{\frac{1}{1+i}}{1 - \frac{1}{1+i}} = \frac{\frac{1}{1+i}}{\frac{1+i-1}{1+i}} = \frac{\frac{1}{1+i}}{\frac{i}{1+i}} = \frac{1}{i}$$

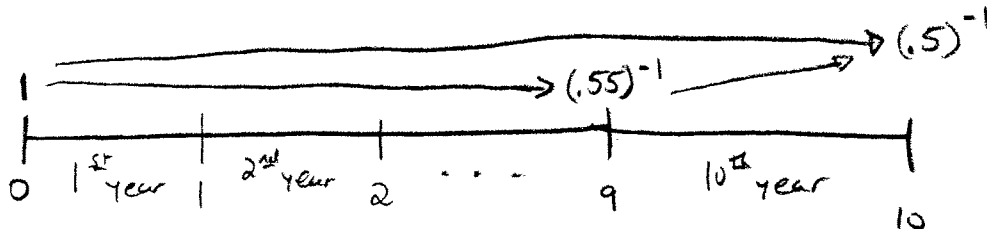
$$\therefore \sum = \frac{1 - v^n}{i}$$

Remark! $1 - v = d = iv$

3. An account credits interest using a simple discount rate of 5% over a 15-year period. During the tenth year, this account is earning interest at what annual effective interest rate?

A. 4% B. 6% C. 8% D. 10% E. 12%

$$a(9) = (1 - .05(9))^{-1} \quad a(10) = (1 - .05(10))^{-1}$$



$$(.55)^{-1} (1+i) = (.5)^{-1} \implies 1+i = \frac{(.5)^{-1}}{(.55)^{-1}}$$

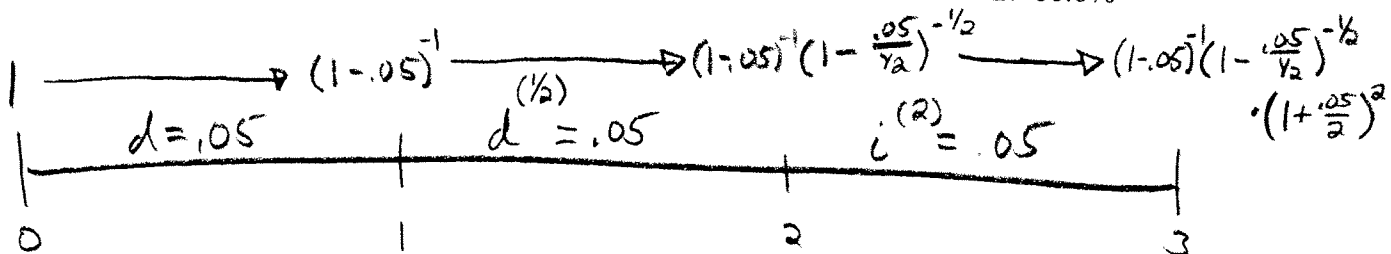
$$\implies i = 10\%$$

4. You are given a loan on which interest is charged over a 3-year period as follows:

- I. An effective rate of discount of 5% for the first year
- II. A nominal rate of discount of 5% compounded every 2 years for the second year
- III. A nominal rate of interest of 5% compounded semiannually for the third year

Calculate the annual effective rate of interest over the 3-year period.

A. 5.0% B. 5.1% C. 5.2% D. 16.0% E. 16.6%



$$\therefore (1+i)^3 = (1-.05)^{-1} \left(1 - \frac{.05}{2}\right)^{-1/2} \left(1 + \frac{.05}{2}\right)^2$$

$$\implies i \doteq 5.2\%$$

5. An investor puts 1000 into Fund X and 1000 into Fund Y. Fund Y earns compound interest at the annual rate of $j > 0$, and Fund X earns simple interest at the annual rate of $1.10j$. At the end of 2 years, the amount in Fund Y is 30 more than the amount in Fund X.

After 5 years, the amount in Fund Y exceeds the amount in Fund X by E . Determine E .

A. 1063

B. 1113

C. 1163

D. 1213

E. 1263

X: $1000 \xrightarrow{\hspace{10em}} 1000(1 + 1.1j(2)) = 1000 + 2200j$

Y: $1000 \xrightarrow{\hspace{10em}} 1000(1+j)^2 = 1000(1+2j+j^2) = 1000 + 2000j + 1000j^2$

$$\therefore 1000 + 2000j + 1000j^2 = 30 + 1000 + 2200j$$

$$1000j^2 - 200j - 30 = 0$$

$$(10j - 3)(100j + 10) = 0$$

$$\underline{j = .3} \quad \text{or} \quad \cancel{j = -.1}$$

After 5 years

$$E = 1000(1.3)^5 - 1000(1 + 1.1(.3)(5))$$

$$\doteq 1063$$