Section 4: General Force of Interest

Relating force of interest to accumulation functions:

Given \(a(t)\), then \(\delta_t = \frac{a'(t)}{a(t)}\) \((t\) is measured in years\)

Given \(\delta_t\), then \(a(t) = e^{\int_0^t \delta_r \, dr}\) \((t\) is measured in years\)

Accumulating and Discounting using General Force of Interest:

\[
Y = X \cdot e^{\int_k^t \delta_r \, dr}, \text{ or equivalently, } X = Y \cdot e^{\int_t^k \delta_r \, dr}
\]

Special Cases:

1. 
\[
\delta_t = c \cdot \frac{f'(t)}{f(t)} \Rightarrow a(t) = \left(\frac{f(t)}{f(0)}\right)^c
\]

2. Constant Force of Interest: \(\delta_t = \delta\) (see earlier notes on continuous compounding)

\[
a(t) = e^{\delta t}
\]
1. Given \( a(t) = 1 + 2t + \frac{1}{2}t^2 \), determine an expression for the general force of interest.

2. Given \( a(t) = 100 + 200t + 50t^2 \), determine \( \delta_2 \).

3. Given \( \delta_t = \frac{6t}{2+6t^2} \) determine \( a(1) \).

4. Suppose \( \delta_t = .02t, t > 0 \).
   a. Determine the accumulation function.
   b. Determine the accumulated value at time 7 of the time 3 value of 100.

5. Given \( \delta_t = \frac{.03}{1-.03t} \) determine the discounted value at time 2 of the time 6 value of 50.
Solutions to Module 1 Section 4 Problems:

1) \[ S_t = \frac{2 + t}{1 + 2t + \frac{3}{4} t^2} \]

2) \[ s_2 = \frac{4}{7} \]

3) \[ a(1) = 2 \]

4) (a) \[ a(t) = e^{-0.1t^2} \]
   (b) \[ X = 149.18 \]

5) \[ X = 43.12 \]