Section 4: Bond Introduction

Bonds are nothing more than loans, but from a different perspective. When we purchase a bond, we are lending money to the issuer of the bond. So we are taking the perspective of the lender rather than the borrower.

Terminology and Notation:

\( P \) – price of the bond (the amount we are lending)
\( i \) – yield rate (as a periodic effective interest rate)
\( F \) – face value (or par value)
\( r \) – coupon rate (as a periodic effective interest rate)
\( Fr \) – coupon amount
\( n \) – number of coupons
\( C \) – redemption value

When we purchase a bond, we pay the price \( P \) today in exchange for receiving the periodic coupon payments of \( Fr \) and the redemption value of \( C \) at time \( n \). The timeline is:

\[
\begin{array}{cccccccc}
0 & 1 & 2 & \cdots & n \\
F_r & F_r & F_r & \cdots & F_r \\
\uparrow & \downarrow & \downarrow & \cdots & \downarrow \\
P & C \\
\end{array}
\]

If the price \( P \) is less than the redemption value \( C \) (i.e., \( P < C \)), we say the bond is bought at a discount. The amount of discount is \( C - P \). If the price \( P \) is more than the redemption value \( C \) (i.e., \( P > C \)) we say the bond is bought at a premium. The amount of premium is \( P - C \). If the price \( P \) is equal to the redemption value \( C \) (i.e., \( P = C \)) we say the bond is bought at a par.

BEWARE:
1. Most bonds are "redeemable at par". This means the redemption value \( C \) is equal to the face value \( F \) (i.e., \( C = F \)). Note the difference between "bought at par" and "redeemable at par". Unless told or implied otherwise, you may assume the bond is redeemable at par.
2. In the problems, the stated rate associated to the coupons is a nominal rate. For example, if the problem states we have a 1000 par value bond with 8% semiannual coupons, then the 8% is a nominal rate compounded semiannually. The coupon rate, \( r \), would be 4% in this case, and so the amount of each coupon would be 1000(0.04) = 40.

Pricing Formulas:

\[
P = Fr a_{\frac{n}{i}} + C v^n \quad \text{(Standard Pricing Formula)}
\]
\[
P = C + (Fr - Ci) a_{\frac{n}{i}} \quad \text{(Premium/Discount Formula)}
\]
Module 3 Section 4 Problems:

1. A 10-year 1000 face value bond with 8% annual coupons is bought to yield 6% annual effective. The purchase price is 1259. Determine the redemption value to the nearest dollar.

2. A 20-year 1000 par value bond, redeemable at par, with 7% semiannual coupons is bought for 901. Determine the annual effective yield for the bond.

3. A 20-year 100 par value bond with 8% quarterly coupons is bought to yield 6% compounded quarterly. Determine whether the bond is bought at a premium or discount, and determine the amount of premium or discount.

4. An n-year bond, redeemable at par, with 6% coupons payable semiannually, is bought to yield 7% compounded semiannually. Determine whether the bond is bought at a premium or discount.

5. A bond has quarterly coupons of 18 and is redeemable for 1200. Determine the yield rate, as a nominal rate compounded quarterly, at which the bond would be bought at par.
Answers to Module 3 Section 4 Problems

1) 1200

2) 8.16%

3) Premium of 23.20

4) Discount

5) 69%