MAP 4170 Name:______________________________

Test 1 Date: September 25, 2012

Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Eli owes Archie payments of 2000 in 1 year and 1000 in 2 years. Eli offers to make a single payment of 2610, immediately, claiming that the total present value of the 2 future payments is 2610. Determine the nominal interest rate compounded monthly $i^{(12)}$ that would make Eli's claim true.

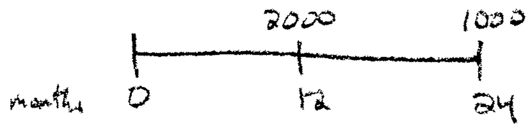
(A) 0.88%

(B) 0.93%

(C) 5.55%

(D) 10.58%

(E) 11.1%



v = monthly discount factor

$$PV = 2610 = 2000v^{12} + 1000v^{24} \quad (\text{quadratic in } v^{12})$$

$a = 1000 \quad b = 2000 \quad c = -2610$

$$v^{12} = \frac{-2 \pm \sqrt{4 + 4(1)(2.61)}}{2(1)} = 0.9$$

i - meir

$$\Rightarrow (1+i)^{12} = \frac{1}{0.9} \Rightarrow i = \left(\frac{1}{0.9}\right)^{1/12} - 1 = \frac{i^{(12)}}{12}$$

$$\Rightarrow i^{(12)} \doteq 10.58\%$$

2. At time 0, Peyton deposits an amount into an account that credits interest using a simple discount rate d. There were no other deposits made into the account. At the end of year 3 there is 1000 in the account and at the end of year 12 there is 1500 in the account. Determine d.

$a(t) = (1 - dt)^{-1}$

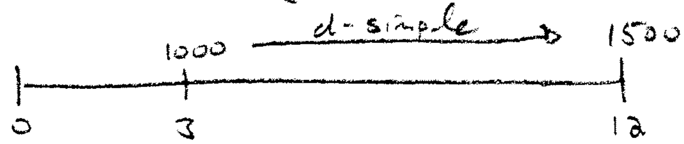
(A) 3.33%

(B) 3.57%

(C) 3.71%

(D) 3.92%

(E) 4.07%



$$1500 = 1000\,\frac{a(12)}{a(3)} = 1000\,\frac{(1-12d)^{-1}}{(1-3d)^{-1}}$$

$$\therefore 1500(1 - 12d) = 1000(1 - 3d)$$

$$\Rightarrow d \doteq 3.33\%$$

3. Betty deposits an amount at time 0 into a fund which credits interest using a simple interest rate i. The force of interest in the account at time 10 is equal to 0.05.

   Charlie deposits 1000 into a separate account in which interest is credited using a nominal interest rate of i, compounded quarterly. Determine the amount in Charlie's account at the end of 10 years.

   (A) 1645

   (B) 2685

   (C) 4065

   (D) 5830

   (E) 7040

$B: a(t) = 1 + it \Rightarrow \delta_t = \frac{i}{1+it}$

$\delta_{10} = .05 \Rightarrow .05 = \frac{i}{1+10i} \Rightarrow i = 0.10$

$C: a(t) = \left(1+\frac{i}{4}\right)^t \quad t = \#$ of quarters

$i = .10$

$AV = 1000\left(1+\frac{.10}{4}\right)^{40} \doteq 2685$

4. Determine which of the following equations represents the correct relationship between a nominal interest rate compounded monthly and a nominal interest rate compounded quarterly.

   (A) $i^{(4)} = 4\left[\left(1+\frac{i^{(12)}}{12}\right)^4 - 1\right]$

   (B) $i^{(4)} = 4\left[\left(1+\frac{i^{(12)}}{12}\right)^{12} + 1\right]$

   (C) $i^{(4)} = 4\left[\left(1+\frac{i^{(12)}}{12}\right)^4 + 1\right]$

   (D) $i^{(4)} = 4\left[\left(1+\frac{i^{(12)}}{12}\right)^3 - 1\right]$

   (E) $i^{(4)} = 4\left[\left(1+\frac{i^{(12)}}{12}\right)^{12} - 1\right]$

$\left[\left(1+\frac{i^{(4)}}{4}\right)^4 = \left(1+\frac{i^{(12)}}{12}\right)^{12}\right]^{1/4}$

$\Rightarrow 1+\frac{i^{(4)}}{4} = \left(1+\frac{i^{(12)}}{12}\right)^3$

$\Rightarrow i^{(4)} = 4\left[\left(1+\frac{i^{(12)}}{12}\right)^3 - 1\right]$

5. A fund credits interest using an interest rate of 10% compounded every other year for the first four years, and a nominal discount rate of 12% compounded monthly thereafter. Determine the accumulated value after 8 years of a deposit of 1000.

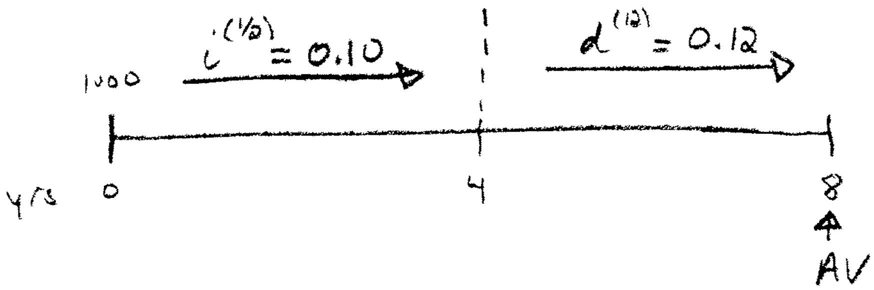
(A) 2320

(B) 2330

(C) 2340

(D) 2350

(E) 2360



$$AV = 1000\left(1 + \frac{i^{(1/2)}}{1/2}\right)^{2}\left(1 - \frac{d^{(12)}}{12}\right)^{-48}$$

$$= 1000(1+0.2)^{2}(0.99)^{-48} \doteq 2332.78$$

6. At time 0, Jason deposits 500 into an account in which the force of interest is $\delta_t = \dfrac{0.5t}{2+t^2}$, for $t > 0$. At the end of year 4, Jason makes an additional deposit of X into the account. The amount of interest earned between years 3 and 5 is 200. Determine X.

(A) 30

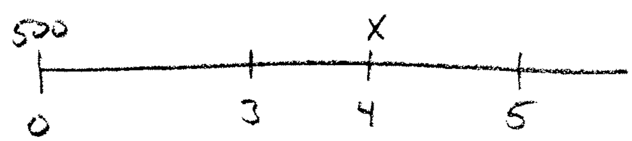
(B) 50

(C) 70

(D) 90

(E) 110

$$\delta_t = \frac{1}{4}\cdot\frac{2t}{2+t^2} \implies a(t) = \left(\frac{2+t^2}{2}\right)^{1/4}$$



Since an additional deposit of X is made at time 4, $I_{[3,5]} = AV_5 - AV_3 - X$

$$AV_5 = 500\,a(5) + X\,\frac{a(5)}{a(4)} = 500\left(\frac{27}{2}\right)^{1/4} + X\left(\frac{27/2}{18/2}\right)^{1/4}$$

$$AV_3 = 500\,a(3) = 500\left(11/2\right)^{1/4}$$

$$\therefore I_{[3,5]} = 200 = 500(13.5)^{1/4} + X(1.5)^{1/4} - 500(5.5)^{1/4} - X$$

$$\implies X \doteq 68.32$$

7. Determine $\frac{d}{dd}(v)$, the derivative of v with respect to d, where d denotes a periodic effective discount rate and v is the corresponding periodic discount factor.

(A) $-v^{-2}$

(B) $-v^{-1}$

(C) -1

(D) $-v$

(E) $-v^{2}$

$$v = 1 - d$$

$$\therefore \frac{d}{dd}(v) = -1$$

8. The force of interest at time t for a certain account is $\delta_t = 0.02t,\ t > 0$. Determine the corresponding annual effective discount rate for year 2 for this account.

(A) 2.96%

(B) 2.99%

(C) 3.02%

(D) 3.05%

(E) 3.08%

$$d_2 = \frac{a(2) - a(1)}{a(2)}$$

$$a(1) = e^{\int_0^1 .02t\,dt} = e^{.01t^2\big|_0^1} = e^{.01}$$

$$a(2) = e^{\int_0^2 .02t\,dt} = e^{.01t^2\big|_0^2} = e^{.04}$$

$$\therefore d_2 = \frac{e^{.04} - e^{.01}}{e^{.04}} \doteq 2.96\%$$

9. Willie deposits 500 into an account that credits interest using a simple discount rate *d* for the first year and then a semiannual effective discount rate of *d* thereafter. At the end of 2 years, the account balance is 1000. Determine *d*.

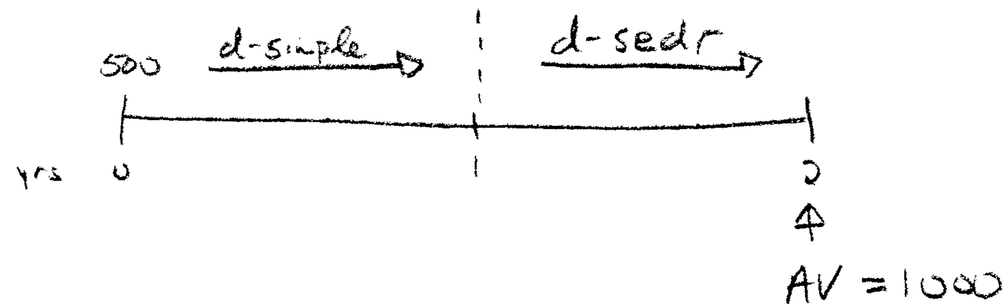
(A) 0.10

(B) 0.15

(C) 0.20

(D) 0.25

(E) 0.30



$$1000 = 500(1-d)^{-1} \cdot (1-d)^{-2} = 500(1-d)^{-3}$$

$$\Rightarrow d \doteq 0.2$$

10. Determine the constant force of interest that is equivalent to an interest rate of 10% compounded quarterly.

(A) 2.38%

(B) 2.47%

(C) 9.35%

(D) 9.53%

(E) 9.88%

$\delta = \ln(1+i)$ $\quad i$ - aeir

$i^{(4)} = 10\% \Rightarrow j = \frac{.10}{4} = .025$ - qeir

$(1+j)^4 = 1+i$

$$\Rightarrow \delta = \ln(1+i) = \ln(1.025)^4$$

$$= 4\ln(1.025) \doteq 9.88\%$$