$\qquad$
Test 4
Date: December 5, 2013
Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. Rob purchased two bonds to form a portfolio as given below:
(i) Bond A is a 20-year bond with coupons at 5\%, a duration of 14 and a purchase price of 1297.
(ii) Bond B is a 20-year bond with coupons at 3\%, a duration of 15.32 and a purchase price of $P$.

For both bonds the duration and price were determined using an annual effective interest rate $i$. Using the same interest rate, the duration of the portfolio is 14.575 . Determine $P$.
A) 950
B) 975
C) 1000
D) 1025
E) 1050
2. You are given the following table of interest rates:

| Calendar Year <br> of Investment | Investment Year Rates |  |  | Portfolio Rates |
| :---: | :---: | :---: | :---: | :---: |
| $Y$ | $i_{1}^{Y}$ | $i_{2}^{Y}$ | $i_{3}^{Y}$ | $i^{Y+3}$ |
| 2002 | 0.05 | 0.06 | 0.07 | 0.04 |
| 2003 | 0.06 | 0.06 | 0.06 | $X \%$ |
| 2004 | 0.08 | 0.08 | 0.10 | $(X+2) \%$ |

1000 is invested on $01 / 01 / 2002$ and another 1000 is invested on $01 / 01 / 2004$. The total amount in the account as of $12 / 31 / 2006$ is 2565 . Determine the portfolio rate for 2007.
A) $3.5 \%$
B) $4.0 \%$
C) $4.5 \%$
D) $5.0 \%$
E) $5.5 \%$
3. On January 1 a pension fund has $\$ 500,000$. The balance on June 30 is $\$ 530,000$ and on July 1 benefit payments of $\$ 30,000$ are paid from the fund. A contribution of $\$ 100,000$ is paid into the fund on date October 1. Immediately before the contribution, the balance in the fund is $\$ 525,000$. The balance on December 31 is $\$ 750,000$. Kevin calculates the time weighted rate of return on the fund to be $K$ and Linda calculates the dollar weighted rate of return on the fund to be $L$. Determine the ratio $K / L$
A) 0.90
B) 0.95
C) 1.00
D) 1.05
E) 1.10
4. A bond is priced at 1412.50 to yield $6 \%$ annual effective, and the Macaulay duration using this interest rate is 13.6 . Determine the price of the bond if the interest rate is changed to $6.5 \%$.
A) Less than 1318
B) Greater than or equal to 1318 but less than 1322
C) Greater than or equal to 1322 but less than 1326
D) Greater than or equal to 1326 but less than 1330
E) Greater than or equal to 1330

For 5. and 6.
Consider a yield curve defined by the equation $i_{k}=.09-(.06)(.94)^{k-1}$ where $i_{k}$ is the annual effective rate of return for zero coupon bonds with maturity of $k$ years.
5. Determine the annual yield on 3-year $5 \%$ annual coupon bonds that is consistent with this yields curve.
A) $3.62 \%$
B) $3.64 \%$
C) $3.66 \%$
D) $3.68 \%$
E) $3.70 \%$
6. Determine the 1-year forward rate for year 3 implied by this yield curve.
A) $4.38 \%$
B) $4.40 \%$
C) $4.42 \%$
D) $4.44 \%$
E) $4.46 \%$
7. Bond A is a 1-year zero-coupon bond with a redemption value of $F_{1}$. It can be bought to yield $10 \%$ annual effective.

Bond B is a 2-year zero-coupon bond with a redemption value of $F_{2}$. It can be bought to yield 5\% annual effective.

Cecil has the following obligations:
$X$ due at the end of 1 year
$X-410$ due at the end of 2 years
Cecil pays a total of 13,300 to purchase enough of each type of bond in order to exactly match the obligations. Determine $X$.
(A) 7,510
(B) 7,530
(C) 7,550
(D) 7,570
(E) 7,590
8. Payments of 100,200 , and 300 are to be made at the end of years 1,2 , and 3 , respectively. Determine the modified convexity of this sequence of payments using an annual effective interest rate of $6 \%$.
(A) 6.67
(B) 6.87
(C) 7.07
(D) 7.27
(E) 7.47
9. Liability payments consist of 1,000 due at the end of 5 years and another 1000 due at the end of 8 years. Assets of amount $\$ X$ due in 3 year and $\$ Y$ due in 10 years have the same present value and duration as the liabilities. Interest is at an annual effective rate of $10 \%$. Determine $X$.
A) 768
B) 778
C) 788
D) 798
E) 808
10. You are given the following information about two bonds that will mature at par value in 10 years:
(i) Bond A has a par value of $10000,5 \%$ annual coupons, and is priced at 10963.
(ii) Bond B has a par value of $1000,4 \%$ annual coupons, and is priced at 1013 .

Determine the 10-year spot rate consistent with the pricing of these bonds.
(A) $3.7 \%$
(B) $3.8 \%$
(C) $3.9 \%$
(D) $4.0 \%$
(E) $4.1 \%$

