MAP 4170
Test 4

Name:______________________________

December 4, 2014

Each problem is worth 10 points. Show sufficient work for full credit.

1. A special 10-year bond with annual coupons has an initial coupon of 50. Each subsequent coupon is 3% greater than its preceding coupon. The redemption value of the bond is 1200. Determine the modified duration of the bond using an annual effective interest rate of 3%.

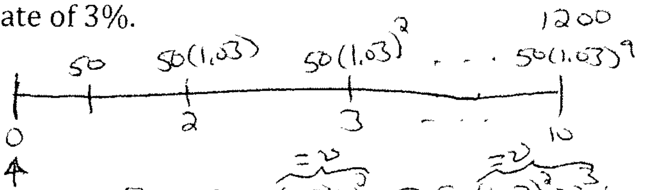
A) 7.93

[✓ B) 8.17]

C) 8.42

D) 8.67

E) 8.93



$v = \frac{1}{1.03}$

$$\text{MacD} = \frac{50v + 2\cdot 50\overbrace{(1.03)v^2}^{=v} + 3\cdot 50\overbrace{(1.03)^2v^3}^{=v} + \cdots + 10\cdot 50\overbrace{(1.03)^9v^{10}}^{=v} + 10(1200)v^{10}}{50v + 50\underbrace{(1.03)v^2}_{=v} + 50\underbrace{(1.03)^2v^3}_{=v} + \cdots + 50\underbrace{(1.03)^9v^{10}}_{=v} + 1200v^{10}}$$

$$= \frac{50v(1+2+3+\cdots+10) + 12000v^{10}}{50v(10) + 1200v^{10}} \doteq 8.415158$$

$$\therefore \text{Mod D} = v \cdot \text{MacD} \doteq 8.17$$

2. Determine the modified convexity of a 20-year 1000 par-value zero-coupon bond, redeemable at par, using an annual effective interest rate of 5%.

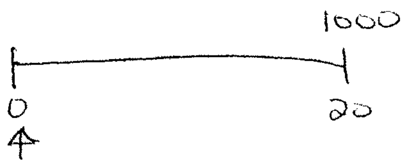
A) 310

B) 345

[✓ C) 380]

D) 420

E) 465



$$P(i) = 1000v^{20} = 1000(1+i)^{-20}$$

$$P'(i) = -20\cdot 1000(1+i)^{-21}$$

$$P''(i) = 21\cdot 20\cdot 1000(1+i)^{-22}$$

$$\text{Mod C} = \frac{P''(i)}{P(i)} = \frac{21\cdot 20\cdot 1000v^{22}}{1000v^{20}} = 21\cdot 20\cdot v_{.05}^2$$

$$\Rightarrow \text{Mod C} \doteq 380.95$$

3. For a given yield curve, the 1-year forward rate two years from now (i.e. from time 2 to time 3) is 3.50% and the 2-year forward rate one year from now (i.e. from time 1 to time 3) is 3.25%. If the 1-year spot rate is 3.00%, determine the 2-year spot rate consistent with these forward rates.

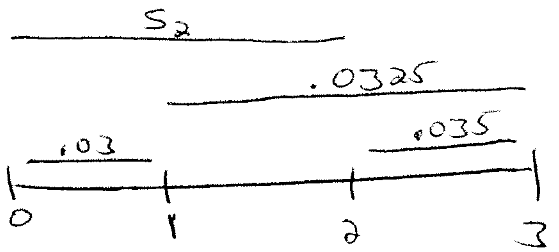
A) 2.95%

[✓ B) 3.00%]

C) 3.05%

D) 3.10%

E) 3.15%



$$(1.03)(1.0325)^2 = (1+S_2)^2(1.035)$$

$$\Longrightarrow S_2 \doteq .03$$

4. A bond that sells for 1031.15 to yield 5% annual effective, sells for 1037.55 to yield *i* annual effective. The bond's Macaulay duration at 5% annual effective is 12.34. Determine *i*.

[✓ A) 4.950%]

B) 4.975%

C) 5.250%

D) 5.050%

E) 5.075%

$$\text{Mod } D = \nu \cdot \text{Mac } D = \frac{12.34}{1.05}$$

$$\Delta P \approx -P \cdot \text{Mod } D \cdot \Delta i$$

$$\Delta P = 1037.55 - 1031.15 = 6.4$$

$$\Delta i = i - .05$$

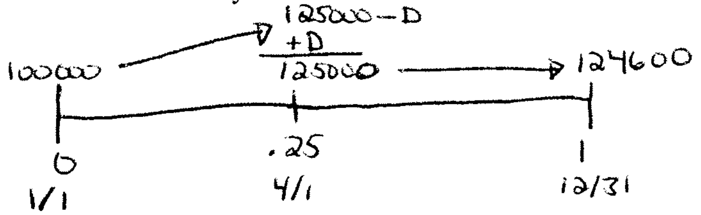
$$P = 1031.15$$

$$\therefore 6.4 = -1031.15 \cdot \frac{12.34}{1.05}(i - .05)$$

$$\Longrightarrow i \doteq .0495$$

5. The balance in an investment account on January 1 is 100,000. A deposit of *D* is made on April 1. The balance in the account immediately after the deposit is 125,000. There are no other transactions within the account during the year, and the balance on December 31 is 124,600.

The dollar weighted return for the year is 4%. Determine the time weighted return for the year.



(A) 4.1%

(B) 4.3%

(C) 4.5%

[✓ (D)] 4.7%

(E) 4.9%

DW: $100000(1.04) + D(1+.04(.75)) = 124600$

$\Rightarrow D = 20000$

$\therefore \quad 1+i_{TW} = \frac{125000-20000}{100000} \cdot \frac{124600}{125000} \Rightarrow i_{TW} = .04664$

6. A liability of 2000 at the end of 1 year and another liability of 50,000 at the end of 2 years are to be exactly matched using a 2-year 1000-par value 5% annual coupon bond, redeemable at par, and a 2-year zero coupon bond. If the yield rate on the coupon bond is 5% and the yield rate on the zero-coupon bond is 4.95%, determine the cost to exactly match the liabilities.

[✓ A)] 47260

B) 47840

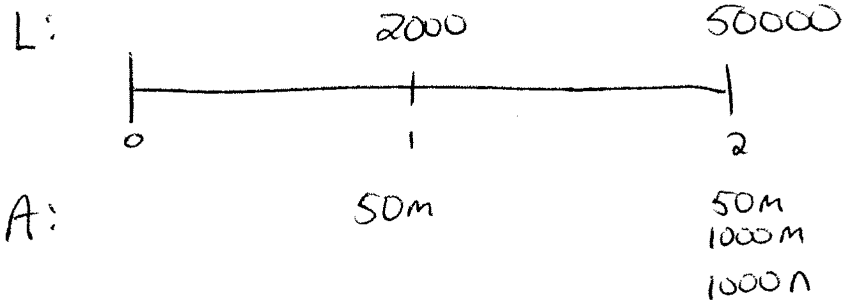
C) 48370

D) 48900

E) 49120

Let m = # of 2-year coupon bonds needed  Fr = 50

n = ——— zero-coupon ———



$\therefore$ exact matching $\Rightarrow$ $\left.\begin{array}{l} 50m = 2000 \\ 1050m + 1000n = 50000 \end{array}\right\} \Rightarrow m = 40,\ n = 8$

$P^{\text{coupon bond}} = 50a_{\overline{2}|.05} + 1000v^2_{.05} \doteq 1000$

$P^{\text{zero-coupon bond}} = 1000v^2_{.0495}$

$\therefore$ total Cost $= 40(1000) + 8(1000v^2_{.0495}) \doteq 47{,}263.15$

7. Liabilities of 3000 and 15000 at the end of years 2 and 3, respectively, are fully immunized at 3% annual effective using 1 and 4 year zero-coupon bonds. Determine the excess of the present value of assets over the present value of liabilities if the interest rate is changed to 1% annual effective.

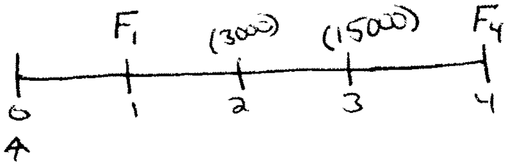
A) 6.1

B) 6.3

C) 6.5

[✓ D) 6.7]

E) 6.9



$$\begin{cases} -\left(F_1 v_{.03} + F_4 v^4_{.03} = 3000 v^2_{.03} + 15000 v^3_{.03}\right) \\ +\left(F_1 v + 4F_4 v^4 = 2\cdot 3000 v^2 + 3\cdot 15000 v^3\right) \end{cases}$$

$$3F_4 v^4 = 3000 v^2 - 30000 v^3$$

$$\Rightarrow F_4 \doteq 11360.9 \quad \text{and} \quad F_1 \doteq 6654.73$$

$$@\ i=.01,\ PV^{Assets} = F_1 v_{.01} + F_4 v^4_{.01} \doteq 17506.44$$

$$\text{and}\ PV^{Liabilities} = 3000 v^2_{.01} + 15000 v^3_{.01} \doteq 17499.74$$

$$\therefore\ \Delta \doteq 6.7$$

8. Tom is planning to buy a car in 5 years. The car he wants costs 30,000 today, and Tom assumes inflation will increase the price of the car by 3% per year. Tom decides to make payments into an account at the beginning of each month for the next five years in order to accumulate the exact amount necessary to buy the car. He assumes that he can earn 7.2% compounded monthly on his deposits. Determine the amount of Tom's monthly deposits.

A) 448

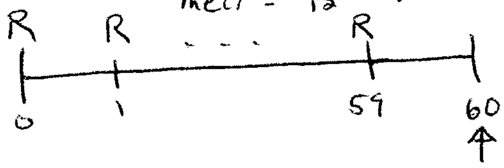
B) 450

C) 464

[✓ D) 480]

E) 483

$meir = \frac{.072}{12} = .006$

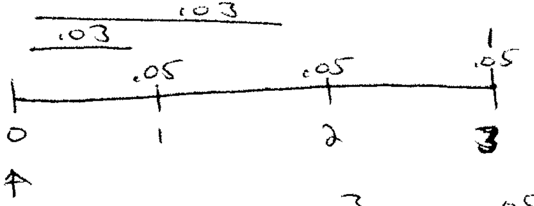


$AV = 30000(1.03)^5$

$$\therefore\ 30000(1.03)^5 = R\,\ddot{s}_{\overline{60}|.6}$$

$$\Rightarrow R = 480.39$$

9. The annual yield rate on 3-year 5% annual coupon bonds, redeemable at par, is 4%. The 1-year and 2-year spot rates are both equal to 3%. Determine the 3-year spot rate that is consistent with the pricing of the bonds. F = 1



A) 3.95%

B) 4.00%

[✓ C)] 4.05%

D) 4.10%

E) 4.15%

$$P = .05\, a_{\overline{3}|.04} + v^{3}_{.04} = \frac{.05}{1.03} + \frac{.05}{(1.03)^2} + \frac{1.05}{(1+s_3)^3}$$

$$\Longrightarrow s_3 \doteq .0405$$

10. You are given the following annual effective interest rates:

| Year | Investment Rates | | Portfolio Rates |
|---|---|---|---|
| $Y$ | $i_1^Y$ | $i_2^Y$ | $i^{Y+2}$ |
| 2008 | 8.00% | 7.75% | 5.00% |
| 2009 | 7.25% | 6.50% | 4.00% |
| 2010 | 6.00% | 6.00% | $i$ |
| 2011 | 6.25% | 5.75% | $i$ |
| 2012 | 4.50% | 4.00% | $i$ |
| 2013 | 4.00% | $i$ | |
| 2014 | 4.50% | | |

Armando deposited 1000 at the beginning 2010 and another 1000 at the beginning of 2011. Beatrice deposited 1100 at the beginning of year 2012 and another 1100 at the beginning of year 2013. At the end of year 2014, Beatrice and Armando have the same accumulated value. Determine the accumulated value at the end of year 2014 of a deposit of 1500 at the beginning of year 2013.

(A) 1580

(B) 1590

[✓ (C)] 1600

(D) 1610

(E) 1620

A: $AV^{EOY}_{2014} = 1000(1.06)^2(1+i)^3 + 1000(1.0625)(1.0575)(1+i)^2$

B: $AV^{EOY}_{2014} = 1100(1.045)(1.04)(1+i) + 1100(1.04)(1+i)$

Setting these AV's equal and cancelling off $(1+i)$ from each term gives the quadratic

$$1000(1.06)^2(1+i)^2 + 1000(1.0625)(1.0575)(1+i) - \big(1100(1.045)(1.04) + 1100(1.04)\big) = 0$$

$$\Longrightarrow 1+i \doteq 1.0271328$$

Answer $= 1500(1.04)(1+i) \doteq 1602.33$