Show sufficient work and clearly mark your answers. Each problem is worth 10 points.

1. A liability of $1,000 due at the end of 2 years is fully immunized at 4% annual effective with 1-year and 3-year zero coupon bonds. Determine the excess of the present value of the assets over the present value of liabilities if the interest rate is changed to 5% annual effective.

\[
\begin{align*}
\text{(A)} & \ 0.088 \\
\text{(B)} & \ 0.0415 \\
\text{(C)} & \ 3.515 \\
\text{(D)} & \ 0.214 \\
\text{(E)} & \ 0.0498 \\
\end{align*}
\]

2. A 30-year bond redeemable at par has 5% annual coupons. Using a 4% annual effective interest rate, determine the Modified Duration of the bond, in years, to the hundredths place.

\[
\begin{align*}
\text{(A)} & \ 16.46 \\
\text{(B)} & \ 17.19 \\
\text{(C)} & \ 16.53 \\
\text{(D)} & \ 15.82 \\
\text{(E)} & \ 17.29 \\
\end{align*}
\]

3. Payment of 100, 200, and 300 are to be made at the end of years 1, 3, and 5 respectively. Determine the convexity of this sequence of payments using an annual effective interest rate of 7%.

\[
\begin{align*}
\text{(A)} & \ 15.89 \\
\text{(B)} & \ 17.27 \\
\text{(C)} & \ 19.83 \\
\text{(D)} & \ 16.01 \\
\text{(E)} & \ 16.31 \\
\end{align*}
\]
4. Consider the yield curve given by the equation \( i_k = 0.06 + 0.005(k - 1) \), where \( i_k \) is the annual effective rate of return on zero coupon bonds with maturity of \( k \) years. Determine \( f_{2,3} \) that is consistent with this yield curve.

\[
\begin{align*}
S_2 &= 0.06 + 0.005 \times 0.06 \\
S_3 &= 0.06 + 0.01 = 0.07
\end{align*}
\]

\[
(1 + 5_2)^2 \times (1 + f_{2,3}) = (1 + 5_3)^3
\]

\[
\Rightarrow \quad 1 + f_{2,3} = \left( \frac{1.08}{1.065} \right)^2 = 1.080071
\]

\[
f_{2,3} = 0.080071
\]

5. On January 1 a pension fund has \( \$X \). The balance on June 30 is \( $530,000 \) and on July 1 benefit payments of \( $30,000 \) are paid from the fund. A contribution of \( $100,000 \) is paid into the fund on date October 1. Immediately before the contribution, the balance in the fund is \( $525,000 \). The balance on December 31 is \( \$Y \). The time weighted rate of return on the fund is \( i_{tw} = 0.3356 \) and the dollar weighted rate of return on the fund is \( i_{dw} = 0.353 \). Find the beginning of the year balance, \( X \), to the nearest thousand.

\[
\begin{align*}
(A) &= 499,000 \\
(B) &= 750,000 \\
(C) &= 500,000 \\
(D) &= 61,000 \\
(E) &= 630,000
\end{align*}
\]

\[
\begin{align*}
\text{(1)} \quad 1.03356 &= \frac{530,000}{B_0} \\
&= \frac{525,000}{500,000} \cdot \frac{B_1}{625,000} \\
\Rightarrow \quad B_1 &= 1.25 B_0
\end{align*}
\]

\[
\text{(2)} \quad B_0 (1.353) - 30,000 \left( 1 + \left( \frac{1}{2} \right)(0.353) \right) + 100,000 \left( 1 + \left( \frac{1}{4} \right)(0.353) \right) = B_1
\]

\[
\Rightarrow \quad 1.353 B_0 - B_1 = -73,530
\]

\[
\Rightarrow \quad B_0 (1.353 - 1.25) = 73,530
\]

\[
B_0 = \frac{500,000,0816}{500,000}
\]

6. A portfolio consists of three assets, A, B, and C. Using an annual effective interest rate of 8%, you are given:

- Asset A has a price of 4000 and modified duration of 5 years.
- Asset B has a price of 3000 and modified duration of \( Y + 1 \) years.
- Asset C has a price of 3000 and modified duration of \( Y \) years.

The Macaulay duration of the portfolio is 21.6 years using an annual effective interest rate of 8%. Determine the Macaulay duration, in years, of Asset B.

\[
\begin{align*}
(A) &= 32.94 \text{ years} \\
(B) &= 28.24 \text{ years} \\
(C) &= 30.5 \text{ years} \\
(D) &= 31.18 \text{ years} \\
(E) &= 31.5 \text{ years}
\end{align*}
\]

\[
\begin{align*}
\text{P}^A &= 4,000 \\
\text{P}^C &= 3,000 \\
\text{P}^B &= \gamma \text{Y} \\
\text{P}^D &= \gamma \text{Y} \\
\text{P}^E &= \gamma \text{Y}
\end{align*}
\]

\[
\begin{align*}
\text{Mac D}^A &= 5 \\
\text{Mac D}^B &= 21.6 \\
\text{Mac D}^C &= \text{P}^A + \frac{\text{Mac D}^B}{1.08} = 20 = \frac{5(4,000)}{10,000} + \frac{(\gamma Y + 1)(3,000)}{10,000} + \frac{\gamma (3,000)}{10,000}
\end{align*}
\]

\[
\Rightarrow \quad 20 = 2 + 0.3(\gamma + 1) + 0.3 \gamma
\]

\[
\Rightarrow \quad \gamma = 5q^2 \text{ and } \gamma + 1 = 30.5 = \text{Mac D}^B
\]

\[
\text{Mac D}^A = (1.08)(30.5) = 32.94 \text{ years}
\]
7. Bond A is a 2-year zero-coupon bond with a redemption value of $F_2$. It can be bought to yield 10% annual effective.
Bond B is a 4-year zero coupon bond with a redemption value of $F_4$. It can be bought to yield 5% annual effective.
Celia has the following obligations:
\[
\begin{align*}
X & \quad \text{due at the end of 2 years} \\
X - 200 & \quad \text{due at the end of 4 years}
\end{align*}
\]
Celia pays a total of $15,000 to purchase enough of each type of bond in order to exactly match the obligations. Determine $X$ to the nearest hundredths place.

\[
\begin{align*}
\sum_{t=2}^{4} F_t = X \\
F_4 = x - 200 \\
L_2 = X \\
15,000 = X \cdot V_{n=2}^2 + (x - 200) \cdot V_{n=4}^6
\end{align*}
\]
\[
\Rightarrow X = \frac{15,000 + 200 (1.05)^{-4}}{(1.1)^2 + (1.05)^{-4}} \approx 9,195.37
\]

(A) 7,274.10
(B) 8,766.99
(C) 6,479.97
(D) 9,195.37
(E) None of the above

8. Paul Sr. wants to buy a vintage Limited Edition Harley Davidson motorcycle in 6 years. The motorcycle currently costs $16,000, and he assumes inflation will increase the price of the Harley by 3% each year. Paul Sr. will make quarterly deposits into an account beginning three months from today in order to have exactly enough to buy the motorcycle outright. Determine the amount Paul Sr. needs to deposit each quarter if he receives 8% compounded quarterly on the deposits.

\[
R \cdot S_{34 \mid 0.02} = 16,000 \cdot (1.03)^6
\]
\[
\Rightarrow R = 628
\]

(A) 264.96
(B) 628
(C) 615.68
(D) 286.15
(E) 720
9. A $1,000 face value 8-year bond with 4% annual coupons, redeemable at $1,200 is selling for $1123.77. A 100 face value 8-year bond with 4% annual coupons, redeemable at par, is selling for $P$. Determine the price, $P$, consistent with an 8-year spot rate of 3.5%.

\[
\begin{align*}
\text{(A)} & 97.2 \\
\text{(B)} & 97.5 \\
\text{(C)} & 92.5 \\
\text{(D)} & 102.5 \\
\text{(E)} & 95.0
\end{align*}
\]

10. You are given the following table of interest rates:

<table>
<thead>
<tr>
<th>Calendar Year or Investment</th>
<th>Interest Year Rate</th>
<th>Portfolio Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>$\beta^Y$</td>
<td>$\beta^Y$</td>
</tr>
<tr>
<td>2009</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>2010</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>2011</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

1000 is invested at the beginning of years 2009 and 2010. The total amount of interest paid for year 2013 is 163.23. Find the portfolio rate for 2014.

\[
\begin{align*}
\text{(A)} & 3.5 \\
\text{(B)} & 5.5 \\
\text{(C)} & 7.5 \\
\text{(D)} & 6.5 \\
\text{(E)} & 7.75
\end{align*}
\]

\[
\begin{align*}
1,000 & \quad 1.07 \quad 1.07 \quad 1.07 \quad \frac{X}{100} \\
1,000 & \quad 1.06 \quad 1.07 \quad 1.08 \quad 1.05 \quad \left(\frac{X}{100}\right) \quad 1,000 \quad (1.07)^3 \quad \left(\frac{X}{100}\right) = 163.23 \\
X_{100} \left[ 1000 \quad (1.06)(1.07)(1.08)(1.05) + 1000 \quad (1.07)^3 \right] & = 163.23 \\
X & = \frac{163.23}{10} \left( (1.06)(1.07)(1.08)(1.05) + (1.07)^3 \right) \\
X & = 6.5 \quad \text{Thus} \quad X + 1\% = 7.5\%
\end{align*}
\]