Show all work for full credit, and use correct notation. Simplify answers completely.

The non-zero transition rates for a 4-state model are:

$$\mu_{x}^{01} = .04$$
$$\mu_{x}^{02} = .02$$
$$\mu_{x}^{21} = .01$$
$$\mu_{x}^{23} = .03$$
$$\mu_{x}^{13} = .001e^{0.1x} = \mu_{x}^{31}$$

Determine

1. $10p_{30}^{12} = 0$

2. Show that $10p_{30}^{00} = e^{-0.6} \approx 0.5488$

$$10p_{30}^{00} = 10p_{30}^{00} = e^{-\int_{0}^{10} (0.04 + 0.02) dt} = e^{-0.06(10)} = e^{-6}$$

3. Show that $10p_{30}^{02} = e^{-0.4} - e^{-0.6} \approx 0.1215$

$$Pr = e_{30}^{oo}, \mu_{30+t}^{2}, \Delta t, 10 \cdot t \cdot p_{30+t}^{22}(Integrand)$$

$$10 \cdot t \cdot p_{30+t}^{22} = 10 \cdot t \cdot p_{30+t}^{22} = e^{-\int_{0}^{10-t} (0.01 + 0.03) dt} = e^{-0.04(10-t)} = e^{-4} \cdot e^{-0.04t}$$

$$\therefore 10p_{30}^{02} = \int_{0}^{10} e^{-0.06t} \cdot e^{-0.04t} dt = \int_{0}^{10} e^{-0.02t} dt = e^{-0.02t} \bigg|_{10} = e^{-0.02(10)} = e^{-0.2} \approx 0.1215$$
You are also given $10p_{30}^{01} \approx 0.2587$ and $10p_{30}^{03} \approx 0.0710$.

4. Determine $10p_{30}^{03} = \text{"rate in"} - \text{"rate out"}$

\[
p_{30}^{03} = \left( t_3 P_{30}^{02} \cdot \mu_{30+t}^{23} + t_3 P_{30}^{01} \cdot \mu_{30+t}^{13} \right) - \left( t_3 P_{30}^{03} \cdot \mu_{30+t}^{31} \right)
\]

\[
= \pm 3 \quad = 0.3 \quad = 0.2587 \quad = 0.001(e^y) \quad \pm 0.110 \quad = 0.001(e^y)
\]

\[
\pm 1.215
\]

\[
10p_{30}^{03} = 0.25(0.3) + 0.2587(0.001(e^y)) - 0.071(0.001(e^y)) = 0.139
\]

5. Use Euler’s Forward Equation with step size equal to 0.2 to approximate $10.2p_{30}^{03}$

\[
\gamma(t+h) = \gamma(t) + h \cdot \dot{\gamma}(t)
\]

\[
t = 10
\]

\[
h = 0.2
\]

\[
10.2p_{30}^{03} = 10p_{30}^{03} + 0.2 \cdot \dot{p}_{30}^{03}
\]

\[
= 0.071 + 0.2(0.139) = 0.07378
\]