Show all work for full credit, use correct notation, and clearly mark your answer.
Use ILT actuarial assumptions (The ILT is posted on the projector.)

A whole life insurance of 5000 payable at the end of the year of death is issued to (30).

1. List out the first three values (the three largest values) of the present value random variable, \( Z \), for this insurance, along with the associated probabilities for each value of \( Z \).

\[
\begin{array}{c|c|c}
Z & 5000 & 5000 \\
0 & 0.0 \times 0 & 1 \\
1 & 1 \times 1 & 2 \\
2 & 2 \times 2 & 3 & \ddots
\end{array}
\]

\[
Z = 5000 \cdot Z_{30}
\]

<table>
<thead>
<tr>
<th>( Z )</th>
<th>( P_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000 ( z_1 ) = 4716.98</td>
<td>( q_{30} = 0.00153 )</td>
</tr>
<tr>
<td>5000 ( z_2 ) = 4449.98</td>
<td>( q_{30} = 0.00161 )</td>
</tr>
<tr>
<td>5000 ( z_3 ) = 4198.10</td>
<td>( q_{30} = 0.00169 )</td>
</tr>
</tbody>
</table>

\[
11 q_{30} = P_{30} \cdot q_{31} = \frac{l_{31} - l_{32}}{l_{30}}
\]

\[
21 q_{30} = 2P_{30} \cdot q_{32} = \frac{l_{32} - l_{33}}{l_{30}}
\]

2. Determine the single net premium for this insurance.

\[
E[Z] = 5000 \cdot E[Z_{30}] = 5000 \cdot A_{30}
\]

\[
= 5000 \cdot (0.10248) = 512.40
\]

3. Determine the variance of the present value random variable for this insurance.

\[
\text{Var}(Z) = 5000^2 \cdot \text{Var}(Z_{30})
\]

\[
= 5000^2 \left[ 2A_{30} - (A_{30})^2 \right]
\]

\[
= 5000^3 \left[ 0.02531 - (0.10248)^2 \right]
\]

\[
= 370196.24
\]
4. Determine the probability that the present value random variable for this insurance is greater than 4750.

\[ Z \text{ is at most } \frac{5000}{1.06} = 4716.98 \ldots \]
\[ \therefore P_r(Z > 4750) = 0 \]

5. Determine the probability that the present value random variable for this insurance is less than 2500.

\[ K \quad Z \quad P_r \]
\[ 2 \geq 2500 \quad 0 \quad \frac{5000}{1.06} \quad 9.30 \]
\[ 2 \frac{2}{2500} \quad 1 \quad \frac{5000}{1.06^2} \quad 11.930 \]
\[ 2 \frac{2}{2500} \quad 10 \quad \frac{5000}{1.06^{11}} \quad 101.930 \]
\[ 2 \frac{2}{2500} \quad 11 \quad \frac{5000}{1.06^{12}} \quad 111.930 \]
\[ Z < 2500 \implies P_r(Z < 2500) = P_r(K \geq 11) \]

\[ Z = 5000 \cdot \left(1.06\right)^{K+1} < 2500 \]
\[ \implies \left(1.06\right)^{K+1} < \frac{1}{2} \]
\[ \implies (1.06)^{K+1} > 2 \]
\[ \implies K > \frac{\ln(2)}{\ln(1.06)} - 1 = 10.8 \ldots \]

\[ P_r(K \geq 11) = \sum_{k=11}^{11} P_3 = \frac{c_{23}}{l_2} = \frac{9287.264}{9501.381} = .97746 \ldots \]
\[ \therefore P_r(Z < 2500) = .97746 \ldots \]