Show all work for full credit, use correct notation, and clearly mark your answer.

1. For a fully continuous 5-year endowment insurance issued to (40), you are given:

   (i) The death benefit is 1000.

   (ii) The pure endowment is 1000

   (iii) \( \mu_x = 0.01 \) and \( \delta_t = 0.04 \)

   (iv) The annual gross premium rate at time \( t \) is \( \pi_t = 45 \)

   (v) Non-settlement expenses are paid continuously at a rate of \( e_t = 10 \)

   (vi) All reserve statements refer to gross premium reserve

   (a) Determine the reserve at time 5.

   (b) Use Thiele’s differential equation to determine an expression for the derivative of the reserve at time \( t \).

   (c) Use Euler’s Forward Equation with \( h = 0.5 \) to determine an approximate value of \( \nu \).

2. For a fully discrete whole life insurance on \( (x) \), you are given:

   (i) The death benefit is 10,000.

   (ii) The withdrawal benefit for year 11, paid at EOY, is 1500.

   (iii) The annual gross premium is 300.

   (iv) Expenses paid at the beginning of year 11 are 3% of gross premium

   (v) \( v = 0.9, \ q^{(d)}_{x+10} = .02, \) and \( q^{(w)}_{x+10} = .10 \)

   (vi) The asset share at time 11 is 230.

Determine the asset share at time 10.
3. For a fully discrete whole life insurance of 1000 on (x), you are given:

(i) Death is the only decrement

(ii) The annual gross premium is 100

(iii) First year expenses are 60% of gross premium, payable at BOY

(iv) \( i = .10 \)

(v) \( q_x = .03 \)

(vi) Reserves at time 0 and time 1 are both equal to 0.

Determine

(a) \( P_r_1 \)

Now suppose the values for interest, expenses, and mortality above are expected values, and that the actual values for interest and mortality are \( i = 0.08 \) and \( q_x = .02 \), respectively, and the actual expenses were equal to the expected expenses.

Determine

(b) the total gain per policy for year 1

(c) the gain from interest prior to the calculation of any other gains by source

(d) the gain from mortality after the gain by interest has been calculated, but prior to the calculation of the gain by expenses.
4. Suppose for a given 3-year product issued to (x), you are given:

(i) the annual premium is 50, payable at the beginning of each year
(ii) the profit vector is \( Pr = (-300, 260, 60, 20) \)
(iii) \( p_x = 0.95 \) and \( p_{x+1} = 0.92 \)
(iv) Profits are discounted using \( i = 0.05 \)

Determine

(a) the discounted payback period
(b) the profit margin
(c) show that the internal rate of return is between 8% and 10% by showing that the net present value when calculated at 8% is positive whereas the net present value when calculated at 10% is negative.

5. For a universal life policy with a face amount of 50,000, you are given:

(i)

<table>
<thead>
<tr>
<th>Policy Year</th>
<th>Annual Premium</th>
<th>Percent of Premium Charge</th>
<th>Annual Expense Charge</th>
<th>Annual COI rate Per 1000</th>
<th>Interest Credited</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3000</td>
<td>7%</td>
<td>10</td>
<td>3</td>
<td>5%</td>
</tr>
</tbody>
</table>

(ii) The account value at the end of year 2 is 5500.

(a) Determine the account value at the end of year 1 if the policy is Type A.

(b) Determine the account value at the end of year 1 if the policy is Type B.
6. Kathy entered a defined benefit plan on 1/1/1990 at age 35 with a salary in 1990 of 50,000. You are given:

(i) The annual retirement benefit is 2% of the final 3-year average salary for each year of service, payable annually beginning at age 65.

(ii) Kathy receives salary increases of 3% every January 1.

(iii) Death is the only pre-retirement decrement.

(iv) Mortality follows the Illustrative Life Table and $i = 0.06$.

Determine, as of the 1/1/2000 valuation date,

(a) the actuarial liability of Kathy’s retirement benefit using the traditional unit credit cost method

(b) the normal cost of Kathy’s retirement benefit using the traditional unit credit cost method

(c) the actuarial liability of Kathy’s retirement benefit using the projected unit credit cost method

(d) the normal cost of Kathy’s retirement benefit using the projected unit credit cost method