Show all work for full credit, use correct notation, and clearly mark your answer.

1. Using ILT assumptions determine
   (a) the single benefit premium for a whole-life insurance of 100,000 issued to (40) with benefit payable at the end of the year of death.
   \[ SBP = 100000 \cdot A_{40}^{\text{ILT}} = 16132 \]
   (b) the variance of the present value random variable for the insurance in part (a)
   \[ V_{\text{ar}}(Z) = 100000^2 \left[ A_{40}^2 - (A_{40})^2 \right]^{\text{ILT}} = 100000^2 \left[ 0.04863 - (16132)^2 \right] = 226058576 \]

2. Using ILT assumptions determine
   (a) the single net premium for a 20-year pure endowment of 5000 issued to (30).
   \[ SNP = 5000 \cdot E_{30}^{\text{ILT}} = 5000 \cdot (29374) = 146870 \]
   (b) the variance of the present value random variable for the insurance in part (a)
   \[ V_{\text{ar}}(Z) = 5000^2 \left[ E_{30}^2 - (E_{30})^2 \right]^{\text{ILT}} = 5000^2 \left[ (0.20) \cdot E_{30}^2 - (29374)^2 \right] = 132658 \]

3. Using ILT assumptions, determine
   (a) the actuarial present value for a 10-year deferred whole-life insurance of 1,000 issued to (35) with benefit payable at the end of the year of death.
   \[ APV = 1000 \cdot A_{101}^{\text{ILT}} \cdot A_{35} = 1000 \cdot A_{10}^{\text{ILT}} \cdot A_{35} = (201.20) \cdot (54318) = 10929 \]
   (b) the variance of the present value random variable for the insurance in part (a)
   \[ V_{\text{ar}}(Z) = 1000^2 \left[ A_{10}^2 \cdot E_{35}^2 - (A_{10} \cdot E_{35})^2 \right] = 1000^2 \left[ (0.6802)(0.54318) - (0.6802)(0.54318)^2 \right] = 8687 \]
4. Using DML(100) mortality and \(i = 0.06\), determine
(a) the EPV for a discrete 10-year term insurance of 10,000 issued to \((40)\).

\[
EPV = 10000 \cdot A_{40:10}^{1,100} = 10000 \left( v_{40} + v_{40}^2 + \cdots + v_{40}^{10} \right)
\]

\[
DML(100) \Rightarrow v_{40} = 0.9495; \cdots; q_{40} = \frac{1}{60}
\]

\[
\therefore EPV = 10000 \cdot A_{40:10}^{1,100} = 10000 \cdot \frac{1}{60} \cdot A_{101,06} = 1226.68
\]

(b) the variance of the present value random variable for the insurance in part (a)

\[
Var(Z) = 10000 \cdot \left( \frac{1}{60} \cdot A_{101,1236} - \left( A_{40:10}^{1,100} \right)^2 \right)
\]

\[
A_{40:10}^{1,100} = \frac{1}{60} \cdot A_{101,06} \Rightarrow \frac{1}{60} \cdot A_{101,1236}
\]

\[
\therefore Var(Z) = 10000 \cdot \left( \frac{1}{60} \cdot A_{101,1236} - \left( \frac{1}{60} \cdot A_{101,06} \right)^2 \right) = 7775.125
\]

5. Using constant force assumptions with \(\mu = 0.02\) and \(i = 0.05\), determine

(a) the EPV for a 20-year deferred whole life insurance of 500 issued to \((x)\) with
benefit payable at the end of the year of death.

\[
EPV = 500 \cdot \omega_{20} A_x = 500 \cdot A_{x+20} \cdot \omega_{20} E_x
\]

\[
A_y = \frac{q_y}{b + i} (\text{for any } y) \quad \omega_{20} E_x = 2^{\omega_{20}} \cdot \omega_{20} P_x = (1.05)^{20} \cdot e
\]

\[
q_x = 1 - p = 1 - e^{-0.02}
\]

\[
\therefore EPV = 500 \left( \frac{1-e^{-0.02}}{1-e^{-0.02} + 0.05} (1.05)^{20} \cdot e^{-4} \right) = 35.83
\]

(b) the variance of the present value random variable for the insurance in part (a)

\[
Var(Z) = 500 \cdot \left( \frac{1}{20} A_x - \left( \frac{1}{20} A_x \right)^2 \right)
\]

\[
A_{x+20} = \frac{q_x}{b + i} \Rightarrow 2 A_{x+20} = \frac{q_x}{b + x + t + i^2} \quad 2 \omega_{20} E_x = 2^{\omega_{20}} \cdot \omega_{20} E_x
\]

\[
\therefore Var(Z) = 500 \cdot \left[ \frac{1-e^{-0.02}}{1-e^{-0.02} + 0.05} (1.05)^{20} \cdot e^{-4} - \left( \frac{1-e^{-0.02}}{1-e^{-0.02} + 0.05} (1.05)^{20} \cdot e^{-4} \right)^2 \right]
\]

\[
= 2570
\]