Show all work for full credit, use correct notation, and clearly mark your answer.

1. (10 points) Using ILT assumptions determine the variance of the present value random variable for a whole-life insurance of 50,000 issued to (35) with benefit payable at the end of the year of death.

\[ Z = 50000 \] 

\[ \text{Var}(Z) = 50000^2 \left[ \hat{A}^{2}_{35} - \left( \hat{A}_{35} \right)^2 \right] \]

\[ = 50000^2 \left[ -0.03488 - (0.12872)^2 \right] = 45,777,904 \]

2. Using ILT assumptions determine

(a) (10 points) the single net premium for a discrete 20-year endowment insurance of 10,000 issued to (40).

\[ \text{SNP} = 10000 \hat{A}_{40:20} = 10000 \left( \hat{A}_{40:20} + \hat{A}_{40:20} \right) \]

\[ \hat{A}_{40:20} = \hat{A}_{40} - 20 E_{40} \cdot \hat{A}_{40} = 16135 - (27414)(.36913) = 0.601 \ldots \]

\[ \hat{A}_{40:20} = 20 E_{40} = .27414 \]

\[ \therefore \text{SNP} = 10000 \cdot (0.601 \ldots + .27414) = 3342.67 \]

(b) (10 points) the variance of the present value random variable for the insurance in part (a).

\[ \text{Var}(Z) = 10000^2 \left[ \hat{A}^{2}_{40:20} - \left( \hat{A}_{40:20} \right)^2 \right] \]

\[ \hat{A}_{40:20} = \frac{\text{see (a)}}{\hat{A}_{40} - 20 E_{40} \cdot \hat{A}_{40} + 20 E_{40}} \cdot 3342.67 \]

\[ 2 \hat{A}_{40:20} = 2 \hat{A}_{40} - 20 E_{40} \cdot 2 \hat{A}_{40} + 20 E_{40} \hat{E}_{40} = (1.06)^{-20} \cdot 0 E_{40} \]

\[ = .04863 - (1.06)^{-20} (0.27414)(.17741) + (1.06)^{-20} (.27414) \]

\[ = .1189 \ldots \]

\[ \therefore \text{Var}(Z) = 10000^2 \left( .1189 \ldots - (.3342)^2 \right) = 720925 \]
3. Using ILT assumptions, determine
   (a) (10 points) the actuarial present value for a 25-year deferred whole-life
       insurance of 1,000 issued to (25) with benefit payable at the end of the year of
death.

\[ Z = 1000 \dot{A}_{25} \Rightarrow APV = 1000 \dot{A}_{25} = 1000 \cdot E_{25} \cdot A_{50} \]

\[ 25 E_{25} = \dot{e}_{25} E_{25} \cdot e_{45} \quad \text{ILT} \quad (0.29873)(0.72988) = 0.218 \cdots \]

(Note: We could also use
\[ 25 E_{25} = \frac{E_{25} \cdot \dot{e}_{30}}{e_{25}} \quad \text{or} \quad 25 E_{25} = \frac{25}{25} P_{25} = (1.06)^{25} \cdot \frac{l_{50}}{l_{25}} \]

\[ \therefore APV = 1000 (0.218 \cdots)(0.24905) = 54.30 \]

(b) (10 points) the variance of the present value random variable for the insurance
    in part (a).

\[ \text{Var}(Z) = 1000^2 \cdot \text{Var}(\dot{A}_{25} Z) = 1000^2 \left[ \frac{2}{\dot{A}_{25}} - (\dot{A}_{25})^2 \right] \]

From above, \( \dot{A}_{25} = 25 E_{25} \cdot A_{50} = (0.218 \cdots)(0.24905) = 0.0543 \cdots \)

Then \( \dot{A}_{25}^2 = \dot{e}_{25} E_{25} \cdot \dot{e}_{50} = (1.06)^{25} \cdot 0.0548 \cdots \)

\[ = (1.06)^{25} \cdot (0.218 \cdots)(0.09476) = 0.0048 \cdots \]

\[ \therefore \text{Var}(Z) = 1000^2 \left[ (0.0048 \cdots) - (0.0543 \cdots)^2 \right] = 186.5 \]