Show all work for full credit, use correct notation., and clearly mark your answer.

For numbers 1 and 2, you are given:

a. \( i = 0.05 \)
b. \( \ddot{a}_{50} = 15 \)
c. \( \ddot{a}_{50} = 0.6 \)

1. Determine the actuarial present value of a 10-year deferred whole life annuity due on (40) with benefit equal to 5000 per month. Use the UDD assumption and note that for \( i = 0.05 \), \( \alpha(12) = 1.000197 \) and \( \beta(12) = 0.46651 \).

\[
\begin{align*}
\text{APV} &= 12(5000) 
\times 10 \cdot \ddot{a}_{40}^{(12)} 
\times 10 \cdot \ddot{a}_{50}^{(12)}
\times E_{40}^{(12)}
\times \ddot{a}_{50}^{(12)}
\times \alpha(12) 
\times \ddot{a}_{50}
\times \beta(12)
\times \alpha(12)
\times \ddot{a}_{50}
\times \beta(12)
\times 1.000197(15)
- 0.46651
\end{align*}
\]

\[
\therefore \text{APV} = 523,312.02
\]

2. Determine the actuarial present value of a 10-year deferred whole life annuity due on (40) with benefit equal to 5000 per month using the two-term Woolhouse Formula.

\[
\begin{align*}
\text{APV} &= 12(5000) 
\times 10 \cdot \ddot{a}_{40}^{(12)} 
\times 10 \cdot \ddot{a}_{50}^{(12)}
\times E_{40}^{(12)}
\times \ddot{a}_{50}^{(12)}
\times \ddot{a}_{50}
\times \frac{2 - e_{24}}{wH}
\times \ddot{a}_{50}
\times \frac{11 - \frac{11}{24}}{24}
\end{align*}
\]

\[
\therefore \text{APV} = 523,500
\]
3. Given $\bar{a}_x = 13$, $\bar{a}_{x:7} = 7$, and $n E_x = 0.6$, determine $\bar{a}_{x+n}$.

$$\bar{a}_x = \bar{a}_{x:7} + n E_x \cdot \bar{a}_{x+n}$$

$$13 = 7 + 0.6 \cdot \bar{a}_{x+n} \implies \bar{a}_{x+n} = 10$$

4. Use constant force assumptions with $\delta = 0.03$ and $\mu = 0.02$ to determine the expected present value of a continuous 10-year temporary annuity issued to $(x)$ paying at a rate of 1000 per year.

$$EPV = 1000 \bar{a}_{x:10\delta} = 1000 \left( \bar{a}_x - 10 E_x \cdot \bar{a}_{x+10} \right)$$

$$\bar{a}_x = \int_0^\infty e^{-\delta t} \cdot e^{-\mu t} \, dt = \int_0^\infty e^{-(\mu+\delta) t} \, dt = \frac{1}{\mu+\delta}$$

$$\bar{a}_{x+10} = \frac{1}{\mu+\delta} \text{ also}$$

$$10 E_x = \int_0^{10} e^{-\delta t} \cdot e^{-\mu t} \, dt = \int_0^{10} e^{-(\mu+\delta) t} \, dt = e^{-(\mu+\delta) \cdot 10}$$

$$\therefore \; EPV = 1000 \left( \frac{1}{\mu+\delta} - e^{-10(\mu+\delta)} \cdot \frac{1}{\mu+\delta} \right) = 1000 \cdot \frac{1}{\mu+\delta} \left( 1 - e^{-10(\mu+\delta)} \right)$$

$$\therefore \; EPV = 1000 \cdot \frac{1}{0.02} \left( 1 - e^{-1.5} \right) = 7869.39$$

Note: $\bar{a}_{x:10\delta} = \int_0^{10} e^{-\delta t} \cdot e^{-\mu t} \, dt$ also.

5. Use constant force assumptions with $\delta = 0.03$ and $\mu = 0.02$ to determine the expected present value of a continuous 10-year certain and life annuity issued to $(x)$ paying at a rate of 1000 per year.

$$EPV = 1000 \bar{a}_{x:10} = 1000 \left( \bar{a}_{10} + 10 E_x \cdot \bar{a}_{x+10} \right)$$

$$10 E_x = e^{-10(\mu+\delta)}$$

$$\bar{a}_{10} = \frac{1 - e^{-10\delta}}{\delta} = \frac{1 - e^{-1.3}}{0.3}$$

$$\bar{a}_{x+10} = \frac{1}{\mu+\delta}$$

$$\therefore \; EPV = 1000 \left( \frac{1 - e^{-1.3}}{0.3} + e^{-5} \cdot \frac{1}{0.05} \right) = 20770$$