Show all work for full credit, use correct notation, and clearly mark your answer.

1. For a fully discrete whole life insurance of 50,000 issued to (35) with annual premiums, you are given:

   (i) Mortality follows the Illustrative Life Table

   (ii) \( i = 0.06 \)

   Show that the premium for which the expected value of the loss-at-issue present value random variable equals 0 is 418.
   \[
   E[L] = 0 \Rightarrow EPV(p) = EPV(b) \Rightarrow \Pi \cdot \ddot{a}_{35} = 50000 \times A_{35}
   \]
   \[
   \therefore \Pi = \frac{50000 \times A_{35}}{\ddot{a}_{35}} \equiv 418 \quad (\approx 418.12)
   \]

2. Using the same actuarial assumptions as in #1, determine the variance of the loss-at-issue present value random variable in #1.

   \[
   oL = 50000 \times Z_{35} - 418 \ddot{Y}_{35} = 50000 \times Z_{35} - 418 \left( \frac{1 - \ddot{Z}_{35}}{\delta} \right)
   \]
   \[
   \Rightarrow oL = (50000 \times \frac{418}{\delta}) \times Z_{35} - \frac{418}{\delta}
   \]
   \[
   \Rightarrow \text{Var}(oL) = (50000 \times 418) [2A_{35} - (A_{35} \ddot{Y}_{35})] \equiv 60298.656
   \]

3. For a fully continuous whole life insurance of 1 issued to \((x)\) with annual premium rate, \( \pi = 0.04 \), you are given:

   (i) \( \mu = 0.04 \)

   (ii) \( \delta = 0.06 \)

   Determine the variance of the loss-at-issue present value random variable,

   \[
   oL = Z_x - \pi \cdot Y_x = Z_x - \pi \left( \frac{1 - \ddot{Z}_x}{\delta} \right) = (1 + \frac{\pi}{\delta}) \times Z_x - \frac{\pi}{\delta}
   \]
   \[
   \Rightarrow \text{Var}(oL) = (1 + \frac{\pi}{\delta})^2 \left[ \frac{\alpha A_x}{\mu + \delta} - (\bar{A}_x)^2 \right]
   \]
   \[
   \bar{A}_x \equiv \frac{\mu}{\mu + \delta} = \frac{1}{10} = 0.1
   \]
   \[
   \Rightarrow \text{Var}(oL) = (1 + \frac{0.04}{0.06})^2 \left[ 0.25 - (0.1)^2 \right] = 0.25
   \]
4. For a fully discrete 2-year endowment insurance on \(x\), you are given

(i) The death benefit is 3000 in year 1 and 2000 in year 2.

(ii) The maturity benefit is 2000.

(iii) The annual premium is 1150.

(iv) \(p_x = 0.75\)

(v) \(d = 0.1\)

Determine the variance of the loss-at-issue present value random variable for this insurance.

\[
\begin{array}{ccc}
0 & 3000 & 2000 \\
0 & 1550 & 750 \\
2 & 2000 & -565 \\
2.402500 & 1550 & .25 \\
3 & 2000 & -565 \\
3.19225 & -565 & .75 \\
\end{array}
\]

\[
\begin{align*}
\text{Var}(oL) &= E[(oL)^2] - (E[oL])^2 \\
&= \Rightarrow \text{Var}(oL) = 838730
\end{align*}
\]

5. For a fully discrete 20-year endowment insurance of 1000 issued to \((40)\), you are given:

(i) The death benefit is paid at the end of the month of death.

(ii) A premium of 3 is paid at the beginning of each month.

(iii) There is a uniform distribution of deaths between integer ages.

(iv) \(2A_{40:20}^{(12)} = 0.12097\)

Using ILT actuarial assumptions, determine the variance of the loss-at-issue present value random variable for this insurance.

\[
\begin{align*}
oL &= 1000 \cdot Z_{40:30}^{(12)} - 3 \cdot \overline{\delta_x^{(12)}} \\
\Rightarrow oL &= (1000 + \frac{36}{\overline{\delta_x^{(12)}}}) \cdot Z_{40:30}^{(12)} - \frac{36}{\overline{\delta_x^{(12)}}} \Rightarrow \text{Var}(oL) = (1000 + \frac{36}{\overline{\delta_x^{(12)}}})^2 \left[ a_{40}^{(12)} - \overline{a_{40}^{(12)}} \right]^2
\end{align*}
\]

\[
\begin{align*}
d^{(12)} &\overset{\text{ILT}}{=} 0.05813 \\
\text{Need} \ A_{40:30}^{(12)}
\end{align*}
\]

Directly:

\[
\begin{align*}
\overline{2A_{40:30}} &= \overline{A_{40:30}} + 2\overline{E_{40}} \\
&\overline{2A_{40:30}} &= \frac{1}{\overline{\delta_x^{(12)}}} \cdot \overline{A_{40:30}} + 2\overline{E_{40}} \overset{\text{ILT}}{=} 3359... \\
\Rightarrow \text{Var}(oL) &= 21342
\end{align*}
\]

Indirectly:

\[
\begin{align*}
\overline{2A_{40:30}} &= \overline{A_{40}} - 2\overline{E_{40}} \cdot \overline{a_{40}^{(12)}} \\
\overline{2A_{40:30}} &= \overline{A_{40}} - 2\overline{E_{40}} \overline{\delta_x^{(12)}} \\
\Rightarrow \text{Var}(oL) &= 21379
\end{align*}
\]