MLC Module 1 Section 9 Exercises

1. Given a 3-state model with $\mu_{X}^{01} = .05, \mu_{X}^{02} = .10, \mu_{X}^{12} = .20$, and all other forces of transition equal to zero, determine

   (a) $5p_{x}^{00}$

   (b) $5p_{x}^{01}$

   (c) $5p_{x}^{02}$

2. Given a 2-state model with $\mu_{x+t}^{01} = .02t$ and $\mu_{x+t}^{10} = 0$, determine

   (a) $10p_{x}^{00}$

   (b) $10p_{x}^{01}$

3. Given a 3-state model with $\mu_{x+t}^{01} = .01 + .02t$ and $\mu_{x+t}^{02} = .02 + .04t$, determine

   (a) $10p_{x}^{00}$

   (b) $nP_{x}^{10}$

   (c) $kP_{x}^{11}$

   (d) $10p_{x}^{02}$

4. Given a 4-state model with $\mu_{X}^{01} = \mu_{X}^{03} = \mu_{X}^{23} = .1, \mu_{X}^{10} = \mu_{X}^{12} = \mu_{X}^{13} = .2$, and all other forces of transition equal to zero, determine

   (a) $0p_{x}^{01}$

   (b) $5p_{x}^{11}$

   (c) $10p_{x}^{22}$

   (d) $p_{x}^{23}$

   (e) $p_{x}^{10}$
5. Given independent lives \((x)\) and \((y)\), where \((x)\) is the husband and \((y)\) is the wife, define the following states of the joint-life, last-survivor process:

State 0: Both Husband and Wife are Alive
State 1: Husband is Dead and Wife is Alive
State 2: Husband is Alive and Wife is Dead
State 3: Both Husband and Wife are Dead

Suppose \(\mu_{01}^{xy} = .01 = \mu_{x}^{23}, \mu_{xy}^{02} = .02 = \mu_{y}^{13}\), and all other forces of transition equal 0.

Determine the probability that at the end of 5 years the husband is dead and the wife is alive.

6. Given a three state model with \(\mu_{x}^{01} = .02, \mu_{x}^{10} = .01, \mu_{x}^{02} = .03 = \mu_{x}^{20}, \mu_{x}^{12} = .04, \mu_{x}^{21} = 0\), you are given \(0.5p_{x}^{00} = .975, 0.5p_{x}^{01} = .010, \text{ and } 0.5p_{x}^{02} = .015\)

(a) determine the value of \(0.5p_{x}^{00}\) according to Kolmogorov differential equations.

(b) use Euler’s method with step size 0.1 to approximate \(0.6p_{x}^{00}\)

7. The non-zero transition rates for a 4-state model are:

\[
\mu_{x}^{01} = .04 \quad \mu_{x}^{02} = .02 \quad \mu_{x}^{21} = .01 \\
\mu_{x}^{23} = .03 \quad \mu_{x}^{13} = .001e^{0.1x} = \mu_{x}^{31}.
\]

(a) Determine \(10p_{30}^{12}\)

(b) Determine \(10p_{30}^{00}\). More generally, determine \(np_{30}^{00}\) for any \(n \geq 0\).

(c) Determine \(10p_{30}^{02}\). More generally, determine \(np_{30}^{02}\) for any \(n \geq 0\).

For parts (d), (e), and (f), you are also given \(10p_{30}^{01} \approx 0.2587\) and \(10p_{30}^{03} \approx 0.0710\).

(d) Determine \(10p_{30}^{01}\) and \(10p_{30}^{02}\)

(e) Use an iteration of Euler’s Forward Equation with step size equal to 0.2 to approximate \(10.2p_{30}^{01}\) and \(10.2p_{30}^{03}\)

(f) Perform another iteration of Euler’s Forward Equation with step size equal to 0.2 to approximate \(10.4p_{30}^{01}\) and \(10.4p_{30}^{03}\)