Module 2 Section 2 Exercises:

For each of the following insurances in Numbers 1 through 10, draw an appropriate timeline and use ILT actuarial assumptions (mortality and interest) to determine

(a) an expression for the present value random variable,
(b) the expected (actuarial) present value of the insurance, and
(c) the variance of the present value random variable.

1. a whole life insurance of 5000, issued to a 35 year old, with the death benefit payable at the end of the year of death
2. a 10-year pure endowment of 350 issued to (40)
3. a 17-year pure endowment of 10000 issued to (35)
4. a discrete 17-year deferred whole life insurance of 500 issued to (35)
5. a discrete 2-year term life insurance of 750 issued to (32)
6. a 17-year term insurance of 2500 issued to (35), with benefit payable at the end of the year of death
7. a 1-year endowment insurance of 25,000 issued to (35), with death benefit payable at the end of the year of death
8. a 17-year endowment insurance of 8000 issued to (35), with death benefit payable at the end of the year of death
9. a discrete whole life insurance of 1000 issued to independent lives aged 30 and 40, with benefit payable upon the first death
10. a discrete whole life insurance of 500 issued to independent lives aged 30 and 40, with benefit payable upon the last death

Note that all the exercises above depend on being able to determine whole life and n-year pure endowment values. So we’ll do some more problems with these products. For Numbers 11 through 13, assume $i = .08$ (note that rates are annual effective interest rates, unless told otherwise) and CF mortality with $\mu = -\ln (0.9)$.

11. For a discrete whole life insurance of 3000 issued to $(x)$, determine

   (a) the EPV of the insurance
   (b) the 2nd raw moment of the present value random variable
   (c) the variance of the present value random variable

12. Determine the APV of a 12-year pure endowment of 100 issued to $(x)$.

13. For a reminder of how to find the EPV of a term policy from whole-life and pure endowments, determine the EPV of a 20-year term insurance of 1000 issued to (40), payable at the end of the year of death.
For Numbers 14 through 16, let's redo similar problems to the previous three with the same \( i = .08 \), but with DML (global UDD) mortality with terminal age 110.

14. For a discrete whole life insurance of 3000 issued to (60), determine

(a) the EPV of the insurance
(b) the 2\(^{nd}\) raw moment of the present value random variable
(c) the variance of the present value random variable

15. Determine the APV of a 12-year pure endowment of 100 issued to (50).

16. Determine the EPV of a 20-year term insurance of 1000 issued to (40), payable at the end of the year of death.

17. In a double decrement model, decrement 1 is death by non-accidental means and decrement 2 is death by accident. You are given:

\[
\begin{array}{|c|c|c|}
\hline
x & q^{(1)}_x & q^{(2)}_x \\
\hline
60 & 0.010 & 0.005 \\
61 & 0.014 & 0.008 \\
\hline
\end{array}
\]

A 2-year term insurance of 10,000 issued to (60), with benefit payable at the end of the year of death, has a double indemnity clause stating that an additional 10,000 will be paid if death occurs by accidental means. Using an annual effective interest rate of 5% determine the actuarial present value of the insurance.

18. You are given:

\[
\begin{array}{|c|c|c|}
\hline
j & 1000A_{x}^{(\text{Dec } j)} & 1000A_{x:nj}^{(\text{Dec } j)} \\
\hline
1 & 148.22 & 417.63 \\
2 & 465.76 & 583.78 \\
\hline
\end{array}
\]

You are also given \( 1000_nE_x = 301.58 \). A discrete whole life insurance issued to (x) pays 2000 if departure occurs within \( n \) years by decrement 1, pays 3000 if departure occurs after \( n \) years by decrement 2, and pays nothing otherwise. Determine the EPV of the insurance.

19. A discrete 3-year term insurance issued to (35) pays 1000 if death occurs in the first year, 750 if death occurs in the second year, and 1250 if death occurs in the third year. Given \( q_{35+k} = 0.02 + 0.005k \), for \( k = 0, 1, \text{ and } 2 \), determine the variance of the present value random variable for the insurance using \( d = 0.05 \).
20. Product A is a discrete 20-year term insurance issued to (30) paying a level benefit of 1000. Product B is a discrete 20-year term insurance offered to the same 30-year old and paying $1000(1.02)^n$ if death occurs in year $n$ ($n = 1, 2, ..., 20$). Using $i = .03$, the APV of Product A is 585. Determine the APV of Product B using $i = 5.06\%$.

21. A discrete 30-year term insurance issued to (50) pays $1000(1.05)^{n-1}$ if death occurs in year $n$. Given $_{30}p_{50} = 0.4$ and $i = .05$, determine the APV of the insurance.

22. Draw the timeline, include the valuation date, that corresponds to the symbol $100(I\bar{A})_x$.

23. A discrete whole life insurance issued to (25) pays 5 if death occurs in the first year, 7 if death occurs in the second year, and so on, where each year’s sum insured is 2 more than the preceding year’s benefit. Write an expression using actuarial notation for the EPV of this insurance.

24. A 10-year term insurance issued to (50), with benefit paid at the end of the year of death, has a sum insured of 1500 for the first year. For subsequent years, the sum insured is 100 less than the previous year’s benefit. Write an expression using actuarial notation for the APV of this insurance.

For Numbers 25 through 28, determine the EPV of the insurance described, using
(a) ILT actuarial assumptions and the UDD assumption between integer ages
(b) ILT actuarial assumptions and the claims acceleration approach

25. A whole life insurance issued to (40) with a benefit of 1000 payable at the end of the quarter of death

26. A 20-year term insurance issued to (40) with a benefit of 1000 payable at the end of the quarter of death

27. A 20-year deferred whole life insurance issued to (40) with a benefit of 1000 payable at the end of the quarter of death

28. A 20-year endowment insurance issued to (40) with a benefit of 1000 payable at the end of the quarter of death