Module 2 Section 5 Exercises:

Note that the same relationships among the different annuities (whole life, n-year deferred, n-year temporary) in the discrete case in section 4 hold in the continuous case in this section. You can review those relationships by looking over Section 4 problems. You’ll see more problems in the end of module quizzes.

1. Given $\bar{a}_x = 5$, $\bar{a}_{x+n} = 4$, and $nE_x = 0.55$, determine $\bar{a}_{x:n|}$. 

For Numbers 2 through 5, draw an appropriate timeline for the annuity described and determine an expression for the present value random variable, $Z$. Then determine the EPV using CF($\mu = .03$, $\delta = .05$) actuarial assumptions.

2. A continuous whole life annuity, issued to $(x)$, paying 5000 per year

3. a continuous 20-year deferred whole life annuity of 1000 issue to an $x$ year old

4. a continuous 20-year temporary annuity of 3000 issued to (60)

5. a continuous 20-year certain-and-life annuity of 2000 issued to (60)

6. Given independent lives $(x)$ and $(y)$ with $\mu_x = .05$, $\mu_y = .15$, and $\delta = .05$, determine

   (a) $\bar{a}_x$, $\bar{a}_y$, $\bar{a}_{xy}$, and $\bar{a}_{x|y}$

   (b) the APV of a continuous annuity that pays 3000 per year while both are alive, 5000 per year to $(x)$ after $(y)$ dies, and 6000 per year to $(y)$ after $(x)$ dies

7. A multi-state model has three states: (0) – Healthy, (1) – Sick, and (2) – Dead. The only forces of transition are:

   $\mu_{01}^{x+t} = 0.04$
   $\mu_{12}^{x+t} = 0.02$
   $\mu_{02}^{x+t} = 0.05$

   Healthy Guy Insurance sells a 20-year annuity that pays continuously at a rate of 9 per year while healthy, 0 otherwise. These annuities are only sold to healthy individuals. Using $\delta = .03$ determine the expected present value of the annuity.
8. A 20-year temporary life annuity issued to (40) pays continuously at an annual rate of \(1000(1.05)^t\) at time \(t\). Determine the APV of this annuity, given \(i \cdot p_{40} = \left(\frac{60-t}{60}\right)^3\) and \(i = .05\).

For Numbers 9 and 10, determine the EPV of the annuity described, using

(a) ILT actuarial assumptions and the UDD assumption between integer ages

(b) ILT actuarial assumptions and the three term Woolhouse approximation

9. A continuous whole life issued to (40) with annual payment rate of 1000

10. A continuous 20-year deferred whole life issued to (40) with annual payment rate of 1000

11. Determine the APV of a continuous 20-year temporary annuity issued to (40) with annual payment rate of 1000, using ILT actuarial assumptions and the UDD assumption between integer ages.

12. Given that mortality follows the ILT, use the three term Woolhouse expression to approximate the difference, \(\dot{e}_{50} - e_{50}\).