Module 4 Section 7 Exercises:

1. For a fully continuous whole life insurance of 100 issued to $(x)$, you are given:

   (i) $\mu_{x+t} = 0.02t$ and $\delta = 0.05$

   (ii) Premiums are determined using the equivalence principle.

   (iii) The annual premium rate for the first year is 5.

(a) Determine the net premium reserve at time 0

(b) Using Thiel’s Differential Equation (TDE), determine the value of the derivative of the reserve, evaluated at time 0, using your answer from part (a)

(c) Use Euler’s Forward Equation with $h = 0.1$, along with your answers from parts (a) and (b), to determine an approximate value of $0.1V$

(d) Using TDE, determine an approximation for the derivative of the reserve, evaluated at time 0.1, using the approximate value of $0.1V$ found in part (c)

(e) Use Euler’s Forward Equation with $h = 0.1$, along with your answers from parts (c) and (d), to determine an approximate value of $0.2V$
2. This is the same set-up as Exercise 1. It is repeated here for convenience.

For a fully continuous whole life insurance of 100 issued to \((x)\), you are given:

(i) \(\mu_{x+t} = 0.02t\) and \(\delta = 0.05\)
(ii) Premiums are determined using the equivalence principle.
(iii) The annual premium rate for the first year is 5.

(a) Determine the net premium reserve at time 0

(b) Using TDE, determine an expression for the derivative of the reserve, evaluated at time 0.1, where the expression depends on the unknown \(0.1V\)

(c) Use Euler’s Backward Equation with \(h = -0.1\), along with your answers from parts (a) and (b), to determine an approximate value of \(0.1V\)

(d) Using TDE, determine an expression for the derivative of the reserve, evaluated at time 0.2, where the expression depends on the unknown \(0.2V\)

(e) Use Euler’s Backward Equation with \(h = -0.1\), along with your answers from parts (c) and (d), to determine an approximate value of \(0.2V\)
3. For a fully continuous 5-year endowment insurance issued to (40), you are given:

(i) The death benefit at time $t$ is $S(t) = 100t$
(ii) The pure endowment is 500
(iii) $\mu_{x+t} = 0.001 \cdot (1.1)^{x+t}$ and $\delta_t = 0.01t$
(iv) The annual gross premium rate at time $t$ is $\pi_t = 2 + t$
(v) Non-settlement expenses are paid continuously at a rate of $e_t = 1 + 0.5t$
(vi) The settlement expense, paid upon death at time $t$, is $10t$
(vii) There is no settlement expense for the pure endowment

(a) Determine the net premium reserve at time 5

(b) Using TDE, determine the value of the derivative of the reserve, evaluated at time 5, using your answer from part (a)

(c) Use Euler’s Backward Equation with $h = -0.5$, along with your answers from parts (a) and (b), to determine an approximate value of $4.5V$

(d) Using TDE, determine an approximation for the derivative of the reserve, evaluated at time 4.5, using the approximate value of $4.5V$ found in part (c)

(e) Use Euler’s Backward Equation with $h = -0.5$, along with your answers from parts (c) and (d), to determine an approximate value of $4V$
4. This is the same set-up as Exercise 3. It is repeated here for convenience.

For a fully continuous 5-year endowment insurance issued to $(40)$, you are given:

(i) The death benefit at time $t$ is $S(t) = 100t$
(ii) The pure endowment is 500
(iii) $\mu_{x+t} = 0.001 \cdot (1.1)^{x+t}$ and $\delta_t = 0.01t$
(iv) The annual gross premium rate at time $t$ is $\pi_t = 2 + t$
(v) Non-settlement expenses are paid continuously at a rate of $e_t = 1 + 0.5t$
(vi) The settlement expense, paid upon death at time $t$, is $10t$
(vii) There is no settlement expense for the pure endowment

(a) Determine the net premium reserve at time 5

(b) Using TDE, determine an expression for the derivative of the reserve, evaluated at time 4.5, where the expression depends on the unknown $4.5V$

(c) Use Euler’s Forward Equation with $h = 0.5$, along with your answers from parts (a) and (b), to determine an approximate value of $4.5V$

(d) Using TDE, determine an expression for the derivative of the reserve, evaluated at time 4.0, where the expression depends on the unknown $4.0V$

(e) Use Euler’s Forward Equation with $h = 0.5$, along with your answers from parts (c) and (d), to determine an approximate value of $4.0V$